READ LESS SCORE MORE

Simplified

Physics

For I PUC

Question and Answers

(2023-24)

Preface

This material is prepared as per reduced syllabus for the year 2023-24. In this material concepts were given in the form of questions and answers so that the young students can be familiarised with question answers which is not given in the text book. Most of the questions were covered in this material except the numerical problems. Students were suggested to refer NCERT text books for complete knowledge.

At last model Questions paper is given as per the blue print suggested by Pre University Board, Bangalore for the year 2023-24 exam.

This material is prepared for the betterment of students and to improve the results and not for commercial purpose. This material is not for sale.

Please provide suggestions to improve this material further. All the best.

Ugranarasimhaswamy
Lecturer in Physics
Mahathma Gandhi Govt P U College, Kunigal.
Tumkur District
Mob-9902856985

CHAPTER-1 UNITS AND MEASUREMENTS

Physical Quantities and units

What is measurement?

It is a process of determining how large or small a physical quantity is as compared to a basic reference standard.

What is a physical quantity? Give examples.

A measurable quantity is called a physical quantity. **Ex:** Length, mass, time, area, volume etc.

What are fundamental quantities?

The physical quantities which are independent of each other are called fundamental quantities.

Mention the fundamental quantities.

There are *SEVEN* fundamental quantities. They are, *Length, Mass, Time, Electric current, Thermodynamic temperature, Amount of substance* and *Luminous Intensity*

What are derived quantities? Give examples.

The physical quantities which can be expressed in the form of a product or quotient of the fundamental quantities are called derived units. **Ex:** Area, Volume, Force, momentum, speed etc.

What is a unit?

The basic, arbitrary chosen, internationally accepted standard of reference which is used to express a physical quantity is called a unit.

What is meant by S I System?

The system of units which is at present internationally accepted for measurement is the **system of International (S I)** and it was developed by General conference on weights and measures in 1971.

Mention the system of units earlier to SI system.

The earlier systems of units are FPS, CGS and MKS system.

What are fundamental units?

The units used to express fundamental quantities are called Fundamental units.

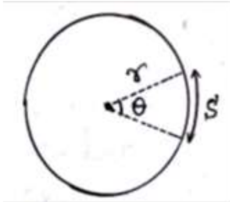
List the fundamental quantities and their units in SI.

Base quantity	SI Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd

Mention the supplimentory quantities and their units SI.

Quantity	S I Units	
	Name	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

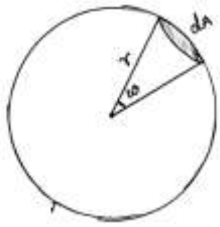
What is plane angle?



It is the ratio of arc length to the radius of the circle.

$$\theta = \frac{s}{r} \quad \text{rad}$$

What is solid angle?



It is the ratio of spherical area enclosed to the square of the radius of the sphere.

$$\omega = \frac{dA}{r^2} \quad sr$$

What are derived units? Give examples.

The units which can be expressed as combination of base units are called derived units. **Ex:** ms^{-1}, ms^{-2}, $kgms^{-1}$, m^2, m^3 etc.

Mention the general guidelines for using symbols and units.

- Symbols for units are written in lower case starting with small letters.
- The unit names are never capitalised, however the unit symbols are capitalised only if the symbol for a unit is derived from a proper name of scientist.
- Symbols for units do not contain any punctual marks and remain unaltered in the plural.

What are the advantages of SI units?

- **It is a rational system**: It uses only one unit for a given quantity.
- **It is a coherent system**: Every unit can be derived from seven fundamental and two supplementary units.
- **It is a metric system**: Multiple and sub multiples of unit can be expressed as the powers of TEN.
- It is internationally accepted.

Keep in mind

Maximum plane angle around point is, $\theta = \frac{2\pi r}{r} = 2\pi$ rad or $\theta = 360^0$

Maximum solid angle at the centre of the sphere is,

$$\omega = \frac{4\pi r^2}{r^2} = 4\,\pi$$

$(i) 360^0 = 2\pi\ rad$

$(ii)\ 180^0 = \pi\ rad$

$(iii)\ \pi\ rad = 180^0$

$1\ rad = \frac{180^0}{\pi} = 57.30$

$1^0 = \frac{\pi}{180^0}\ rad$

$(iv)\ 60' = 1^0$

$1' = \left(\frac{1}{60}\right)^0$

$1' = \left(\frac{1}{60}\right)\frac{\pi}{180}\ rad$

$1' = 2.91 \times 10^{-4}\ rad$

$(v)\ 60'' = 1'$

$1'' = \left(\frac{1}{60}\right)'$

$1'' = \left(\frac{1}{60}\right) \times 2.9 \times 10^{-4}\ rad$

$1'' = 4.85 \times 10^{-6}\ rad$

List the common SI prefixes.

Multiples and Submultiples	Prefixes	Symbols
1 000 000 000 000= 10^{12}	tera	T
1000 000 000= 10^{9}	giga	G
1 000 000= 10^{6}	mega	M
1000= 10^{3}	kilo	k
100= 10^{2}	hecto	h
10= 10^{1}	deka	da
Base unit: 1= 10^{0}		
0.1= 10^{-1}	deci	d
0.01= 10^{-2}	centi	c
0.001= 10^{-3}	milli	m
0.000 001= 10^{-6}	micro	μ
0.000 000 001= 10^{-9}	nano	n
0.000 000 000 001= 10^{-12}	pico	p
0.000 000 000 000 001= 10^{-15}	femto	f
0.000 000 000 000 000 001= 10^{-18}	atto	a

Significant figures

What are significant figures?

In a measured value the reliable digits and the first uncertain digit are known as significant figures.

What is the importance of significant figures?

Significant figures indicate the precision of the instrument. Number of significant figure does not change if we measure a physical quantity in different units.

Mention the rules for determining the significant figures with examples.

- All non-zero digits are significant.
  - Ex: 2341 m → has 4 significant figures.
  - 14.3m → has 3 significant figures

- All the zeros between two non-zero digits are significant.
  - Ex: 308 m → has 3 significant figures.
  - 23.08 m → has 4 significant figures.

- In a number without decimal point trailing or terminal zeros are NOT significant.
  - Ex: 12300 m → has 3 significant figures.
  - 104000 m → has 3 significant figures.

- In number with decimal point, trailing or terminal zeros are significant.

* Ex: 4.700 m → has 4 significant figures.
* 23.04000 m → has 7 significant figures.

- If the number is less than 1, then zeros on the right of decimal point but to the left of the first non-zero digit are NOT significant.

* Ex: 0.067 m → has 2 significant figures.
* 0.0003080 m → has 4 significant figures.

Explain scientific notation method of finding the number of significant figures with example.

In this notation, every number is expressed as $a \times 10^b$, where a is a number between 1 and 10 is called base number and b is any positive or negative exponent of 10. The power of 10 is irrelevant to the determination of significant figures. However all zeros appearing in the base number in the scientific notation are significant.

Ex: 4.700$\times 10^2 m$ has 4 significant figures.

4.700$\times 10^{-3} m$ has 4 significant figures.

Mention rules for arithmetic operations with significant figures with examples.

(i) When numbers are added or subtracted, the number of decimal places in the final result should be equal to the smallest number of decimal places of any term.

Ex: (a) 436.32 + 227.2 = 663.5 (but not 663.52)

(b) 0.3074 – 0.304 = 0.003 (but not 0.0034)

(ii) In multiplication or division, the number of significant figures in the final result should be equal to the number of significant figures in the quantity having the smallest number of significant figures.

Ex: (a) 1.21 ×0.12 = 0.14 (but not 0.1452)

(b) $\frac{5.74}{1.2} = 4.8$ (but not 4.7833)

Mention the rules for rounding of the uncertain digits with examples.

(i) If the digit to be dropped in a number is less than 5, then the preceding digit remains unchanged.

Ex: 1.344 is rounded as 1.34

(ii) If the digit to be dropped in a number is greater than 5, then the preceding digit is raised by 1.

Ex: 1.346 is rounded as 1.35

(iii) If the digit to be dropped in a number is 5, then

(a) the preceding digit remains unchanged if it is EVEN.

Ex: 1.345 is rounded as 1.34

(b) the preceding digit is raised by 1, if it is ODD.

Ex: 1.375 is rounded as 1.38

Dimensions of a physical quantity

What are dimensions of physical quantity? Give example.

Dimensions of a physical quantity are the power to which the base quantities are raised to represent the physical quantity.

Ex: Dimensions of force are MLT^{-2} . Hence force has one dimension in mass, one dimension in length and -2 dimensions in time.

List the symbols for dimensions of fundamental quantities.

Base quantity	Symbol for its dimension
Length	[L]
Mass	[M]
Time	[T]
Current	[A]
Thermodynamic temperature	[K]
Luminous intensity	[cd]
Amount of substance	[mol]

What is dimensional formula? Give examples.

Expression of physical quantity in terms of the base quantities is called dimensional formula.

Ex: Dimensional formula of volume is $[M^0L^3T^0]$, Dimensional formula of Speed is $[M^0LT^{-1}]$

What is dimensional equation? Give examples.

Equation obtained by equating a physical quantity with its dimensional formula is called dimensional equation.

Ex: $[F] = [MLT^{-2}]$, $[V] = [M^0L^3T^0]$

Explain the different types of variables with examples.

(a) Dimensional variable: The physical quantities which possess dimensions and have variable values are called dimensional variables.

Ex: Area, volume, speed, velocity, acceleration, momentum, force etc.

(b) Dimensionless variables: The physical quantities which have no dimensions but have variable values are called dimensionless variables.

Ex: Angle, specific gravity, strain, $sin\ \theta$, $cos\ \theta$, $tan\ \theta$ etc.

Explain the different types of constants with examples.

(a) Dimensional constants: The physical quantities which possess dimensions and have constant values are called dimensional constants.

Ex: Planck's constant, Gravitational constant, speed of light in vacuum etc.

(b) Dimensionless constants: The physical quantities which do not have dimensions but have constant values are called dimensionless constants.

Ex: π, e, pure numbers like 1, 2, 3….etc.

What is dimensional analysis?

The process of examination of dimensions of various physical quantities involved in a relation is called dimensional analysis.

State the principle of homogeneity.

The dimensions of all the terms in an equation must be identical. This principle is called the principle of homogeneity.

Mention the uses of Dimensional analysis.

By dimensional analysis we can,

(i) Check the correctness of an equation.
(ii) Deduce relations between physical quantities.
(iii) Convert the unit of a physical quantity from one system to another.

Check the correctness of the following equation by dimensional analysis.

(i) $v = v_0 + at$

$$v = velocity = \frac{displacement}{time}$$

$$[v] = \frac{[L]}{[T]} = [LT^{-1}]$$

$$v_0 = velocity$$

$$[v_0] = [LT^{-1}]$$

$$at = acceleration \times time$$

$$at = \frac{velocity}{time} \times time$$

$$at = velocity$$

$$[at] = [LT^{-1}]$$

The dimensions of each term on both sides of the equation are the same. Thus equation is dimensionally correct.

(ii) $x = v_0 t + \frac{1}{2} at^2$

$$x = displacement$$

$$[x] = [L]$$

$$v_0 t = velocity \times time$$

$$v_0 t = \frac{displacement}{time} \times time = displacement$$

$$[v_0 t] = [L]$$

$$at^2 = acceleration \times time^2$$

$$at^2 = \frac{velocity}{time} \times time^2$$

$$at^2 = velocity \times time = \frac{displacement}{time} \times time$$

$$at^2 = displacement$$

$$[at^2] = [L] \qquad \left(\because \tfrac{1}{2} \text{ is constant and dimension less}\right)$$

The dimensions of each term on both sides of the equation are the same. Thus equation is dimensionally correct.

(iii) $v^2 = v_0^2 + 2ax$

$$v^2 = (velocity)^2$$

$$v^2 = \frac{(displacement)^2}{(time)^2}$$

$$[v^2] = \frac{[L]^2}{[T]^2} = \frac{[L^2]}{[T^2]} = [L^2T^{-2}]$$

$$v_0{}^2 = (velocity)^2$$

$$[v^2] = [L^2T^{-2}]$$

$$ax = acceleration \times displacement$$

$$ax = \frac{velocity}{time} \times displacement$$

$$ax = velocity \times \frac{displacement}{time} = velocity \times velocity$$
$$ax = (velocity)^2$$
$$[ax] = [L^2T^{-2}] \qquad (\because\ 2\ is\ constant\ and\ dimension\ less)$$

The dimensions of each term on both sides of the equation are the same. Thus equation is dimensionally correct.

The time period of oscillation of a simple pendulum(T) depends on its length(l), mass of the bob(m) and acceleration due to gravity(g). Derive the expression for its time period using the method of dimension.

Let $T \propto l^a m^b g^c$

$T = k\ l^a m^b g^c$ (where k is constant and dimensionless)

$[T] = [time] = [M^0L^0T]$

$[l^a] = [(length)^a] = [L^a]$

$[m^b] = [(mass)^b] = [M^b]$

$[g^c] = [(accelearation\ due\ to\ gravity)^c] = [(LT^{-2})^c] = [L^cT^{-2c}]$

Then, from the principle of homogeneity,

$[M^0L^0T] = [L^a][M^b][L^cT^{-2c}]$

$[M^0L^0T] = [L^aL^cM^bT^{-2c}]$

$[M^0L^0T] = [M^bL^{a+c}T^{-2c}]$

Comparing the exponents on both sides, we have

$b = 0$

$a + c = 0$

$-2c = 1$

On solving the above equations,

$$-2c = 1 \Rightarrow\ c = -\frac{1}{2}$$

and, $a + c = 0$

$$a - \frac{1}{2} = 0\ \Rightarrow\ a = \frac{1}{2}$$

Now substituting the values of $a, b, and\ c,$ in the equation $T = k\ l^a m^b g^c$

$T = k\ l^{1/2} m^0 g^{-1/2}$

$$\boldsymbol{T = k\sqrt{\frac{l}{g}}}$$

The centripetal force(F) acting on a particle moving in a circle depends upon mass(m), velocity(v) and radius of the circle(r). Derive an expression foe centripetal force using the method of dimensions.

Given, $F \propto m^a v^b r^c$

$F = k\ m^a v^b r^c$ (where k is dimensionless constant)

$[F] = [force] = [MLT^{-2}]$

$[m^a] = [(mass)^a] = [M^a]$

$[v^b] = [(velocity)^b] = [(LT^{-1})^b] = [L^bT^{-b}]$

$[r^c] = [(radius)^c] = [L^c]$

Then, from the principle of homogeneity,

$[MLT^{-2}] = [M^a][L^bT^{-b}][L^c]$

$[MLT^{-2}] = [M^aL^bL^cT^{-b}]$

$[MLT^{-2}] = [M^aL^{b+c}T^{-b}]$

On comparing, $a = 1$

$b + c = 1$

$-b = -2$

Solving for b and c, we have

$-b = -2 \implies b = 2$

$b + c = 1$

$2 + c = 1 \implies c = 1 - 2 = -1$

Now substituting the values of $a, b, and\ c,$ in the equation F= $k\ m^a v^b r^c$

$$F = k\ m^1 v^2 r^{-1}$$

$$\boldsymbol{F = k\ \frac{mv^2}{r}}$$

Mention the limitations of dimensional analysis.

1) Dimensionally correct equation need not be actually correct.
2) Correctness of the constants appearing in an equation cannot be verified.
3) Equations involving trigonometric and exponential functions cannot be verified.
4) An equation can be derived only if it is of product type.
5) While deriving en equation the value of constant of proportionality cannot be obtained.
6) This method works only if there are as many equations available as there are unknowns.

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	—	Metre or m	M^0LT^0
Area	A	$\ell \times b$	$(Metre)^2$ or m^2	$M^0L^2T^0$
Volume	V	$\ell \times b \times h$	$(Metre)^3$ or m^3	$M^0L^3T^0$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	M^0LT^{-1}
Momentum	p	p = mv	kgm/s	MLT^{-1}
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	M^0LT^{-2}
Force	F	F = ma	Newton or N	MLT^{-2}
Impulse	–	F × t	N.sec	MLT^{-1}
Work	W	F. d	N.m	ML^2T^{-2}
Energy	KE or U	K.E. = $\frac{1}{2}mv^2$ P.E. = mgh	Joule or J	ML^2T^{-2}
Power	P	$P = \frac{W}{t}$	watt or W	ML^2T^{-3}
Density	d	d =mass/volume	kg/m^3	$ML^{-3}T^0$

Quantity	symbol	formula	S I unit	D F
Pressure	P	$P = F/A$	Pascal or Pa	$ML^{-1}T^{-2}$
Torque	τ	$\tau = r \times F$	N.m.	$ML^{2}T^{-2}$
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$M^{0}L^{0}T^{0}$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/sec	$M^{0}L^{0}T^{-1}$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/sec^2	$M^{0}L^{0}T^{-2}$
Moment of Inertia	I	$I = mr^2$	kg-m^2	$ML^{2}T^{0}$
Angular momentum	J or L	$J = mvr$	$\frac{kgm^2}{s}$	$ML^{2}T^{-1}$
Frequency	ν or f	$f = \frac{1}{T}$	hertz or Hz	$M^{0}L^{0}T^{-1}$
Stress	—	F/A	N/m^2	$ML^{-1}T^{-2}$
Strain	—	$\frac{\Delta\ell}{\ell};\frac{\Delta A}{A};\frac{\Delta V}{V}$	—	$M^{0}L^{0}T^{0}$
Youngs modulus (Bulk modulus)	Y	$Y = \frac{F/A}{\Delta\ell/\ell}$	N/m^2	$ML^{-1}T^{-2}$
Surface tension	T	$\frac{F}{\ell}$ or $\frac{W}{A}$	$\frac{N}{m};\frac{J}{m^2}$	$ML^{0}T^{-2}$
Force constant (spring)	k	$F = kx$	N/m	$ML^{0}T^{-2}$
Coefficient of viscosity	η	$F = \eta\left(\frac{dv}{dx}\right)A$	kg/ms(poise in C.G.S)	$ML^{-1}T^{-1}$
Gravitational constant	G	$F = \frac{Gm_1m_2}{r^2}$ $\Rightarrow G = \frac{Fr^2}{m_1m_2}$	$\frac{N-m^2}{kg^2}$	$M^{-1}L^{3}T^{-2}$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$M^{0}L^{2}T^{-2}$
Temperature	θ	—	Kelvin or K	$M^{0}L^{0}T^{0}\theta^{+1}$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	$ML^{2}T^{-2}$
Specific heat	S	$Q = m \times S \times \Delta t$	$\frac{\text{Joule}}{\text{kg.Kelvin}}$	$M^{0}L^{2}T^{-2}\theta^{-1}$
Latent heat	L	$Q = mL$	$\frac{\text{Joule}}{\text{kg}}$	$M^{0}L^{2}T^{-2}$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1-\theta_2)t}{d}$	$\frac{\text{Joule}}{\text{m sec K}}$	$MLT^{-3}\theta^{-1}$

Quantity	symbol	formula	S I unit	D F
Universal gas constant	R	PV = nRT	$\frac{\text{Joule}}{\text{molK}}$	$ML^2T^{-2}\theta^{-1}$
Mechanical equivalent of heat	J	W = JH	—	$M^0L^0T^0$
Charge	Q or q	$I = \frac{Q}{t}$	Coulomb or C	M^0L^0TA
Current	I	—	Ampere or A	$M^0L^0T^0A$
Electric permittivity	ε_0	$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r^2}$	$\frac{(\text{coul.})^2}{\text{N.m}^2}$ or $\frac{C^2}{N-m^2}$	$M^{-1}L^{-3}A^2T^4$
Electric Potential	V	$V = \frac{\Delta W}{q}$	Joule/coul	$ML^2T^{-3}A^{-1}$
Intensity of electric field	E	E =	N/coul.	$MLT^{-3}A^{-1}$
Capacitance	C	Q = CV	Farad	$M^{-1}L^{-2}T^4A^2$
Dielectric constant or relative permittivity	ε_r	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	—	$M^0L^0T^0$
Resistance	R	V = IR	Ohm	$ML^2T^{-3}A^{-2}$
Conductance	S	$S = \frac{1}{R}$	Mho	$M^{-1}L^{-2}T^{-3}A^2$
Specific resistance or resistivity	ρ	$\rho = \frac{RA}{\ell}$	Ohm × meter	$ML^3T^{-3}A^{-2}$
Conductivity or specific conductance	σ	$\sigma = \frac{1}{\rho}$	Mho/meter	$M^{-1}L^{-3}T^3A^2$
Magnetic induction	B	$F = qvB\sin\theta$ or F = BIL	Teslaorweber/m^2	$MT^{-2}A^{-1}$
Magnetic flux	φ	$e = \frac{d\phi}{dt}$	Weber	$ML^2T^{-2}A^{-1}$
Magnetic intensity	H	$B = \mu H$	A/m	$M^0L^{-1}T^0A$
Magnetic permeability of free space or medium	μ_0	$B = \frac{Idl\sin\theta}{r^2}$	$\frac{N}{amp^2}$	$MLT^{-2}A^{-2}$
Coefficient of self or Mutual inductance	L	$e = L \cdot \frac{dI}{dt}$	Henery	$ML^2T^{-2}A^{-2}$
Electric dipole moment	p	$p = q \times 2\ell$	C.m.	M^0LTA
Magnetic dipole moment	M	M = NIA	amp.m^2	$M^0L^2AT^0$

CHAPTER-2 MOTION IN A STRAIGHT LINE

Know the terms and concepts

What is mechanics?
Mechanics is the oldest and fundamental branch of physics which deals with the study of the state of rest as well as the state of motion of object under the action of force. The study of mechanics is broadly classified in to (i) Statics and (ii) Dynamics

What is static?
It deals with bodies at rest under the action of system of force.

What is Dynamics?
It deals with motion of a body under the action of force.
Dynamics is divided into (a) Kinematics and (b) Kinetics

What is Kinematics?
It deals with the description of motion without reference to the cause of motion.

What is Kinetics?
It deals with what moves and what causes motion.

What is a particle?
A particle is ideally just a piece or quantity of matter, having no linear dimensions but only position and mass.

What is an event?
An event is a physical process that occurs at a point in space and at an instant of time.

Who is an observer?
A person or equipment which can locate, record, measure and interpret an event is called an observer.

What is frame of reference? Explain.
It is the reference in which an observer sits and makes the observations. In order to specify the position, we need to use a reference point and set of axes. The choice of set of axes in a frame of reference depends on the situation.

Motion

What is motion?
Motion is change in position of an object with time.

What is rectilinear motion? Give examples.
Motion of objects along a straight line is called rectilinear motion.
Ex: A car moving along a straight road, a freely falling body.

What is rest?

A body is said to be at rest when it does not changes its position with time.

What is required to specify the position of an object?

To specify the position of an object, a reference point called origin is required.

What is meant by uniform motion?

If an object moving along the straight line covers equal distances in equal intervals of time, then it is said to have uniform motion.

Illustration:

What is the state of a person sitting in a moving bus with respect to a person standing at the bus stop and why?

For the person standing at the bus stand, the person who is inside the moving bus is in motion because the position of the person inside the bus is changing with time with respect to him.

Define velocity?

Velocity is defined as the rate of change of displacement of a body.

$$Velocity = \frac{displacement}{time\ taken}$$

$$\boldsymbol{v = \frac{x}{t}}$$

Write the SI unit and dimensions of velocity.

SI unit is $eter\ per\ second\ (ms^{-1})$. Dimensions are M^0LT^{-1}

Write the characteristics of velocity.

* Velocity is a vector quantity.
* Velocity may be positive, negative or zero.

Define average velocity?

The average velocity of a particle in motion is defined as the ratio of total displacement to the total time taken.

$$Average\ velocity = \frac{total\ displacement}{total\ time\ taken}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Define instantaneous velocity?

Velocity is defined as the limit of average velocity as the time interval Δt becomes infinitesimally small.

$$\boldsymbol{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Keep in mind

* Instantaneous velocity is also called velocity.
* In position-time graph, instantaneous velocity at a point is the slope to the tangent drawn to the curve at that point.

* A vehicle travels half the distance L with speed $V_1$and the other half with speed V_2 then its average speed is,
  $V = \frac{2V_1V_2}{V_1 + V_2}$

What is uniform velocity?

If equal changes of displacement take place in equal intervals of time is called uniform velocity.

Define instantaneous speed (speed)?

It is defined as the magnitude of the instantaneous velocity at that instant.

Keep in mind

When a body moves with uniform velocity neither the magnitude nor the direction of the velocity changes.

Define acceleration?

It is defined as rate change of velocity of a particle.

$$Acceleration = \frac{change\ in\ velocity}{time\ taken}$$

$$\boldsymbol{a = \frac{v - v_0}{t}}$$

Write the SI unit and dimensions of acceleration.

SI unit is $metre\ per\ square\ of\ second$ (ms^{-2}) and dimensions are M^0LT^{-2}

Mention the characteristics of acceleration.

* Acceleration is a vector quantity.
* Acceleration can be positive, negative or zero.

Keep in mind

* Since velocity is a quantity having both magnitude and direction, Acceleration may result from a change in magnitude or a change in direction or changes in both.
* The negative acceleration is called retardation or deceleration.
* If the velocity is zero at an instant, the acceleration need not be zero at that instant as in the case of motion under gravity at the topmost point.
* It is not possible to have constant velocity and variable acceleration.

Define average acceleration?

It is defined as the total change in velocity divided by the total time taken.

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Define instantaneous acceleration?

It is defined as the limit of the average acceleration as the time interval Δt becomes infinitesimally small.

$$\boldsymbol{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}}$$

What is uniform acceleration?

If the velocity of a body changes by equal amount in equal intervals of time, however small these time intervals may be is called uniform acceleration.

What is the acceleration of a body moving with constant velocity?

Zero.

Illustration:

Velocity of a body changes from $10\, ms^{-1}$ to $15\, ms^{-1}$ in $5\, second$. Calculate the average acceleration.

$$\bar{a} = \frac{v_2 - v_1}{\Delta t} = \frac{15 - 10}{5} = \frac{5}{5} = 1\, ms^{-2}$$

A car travels with a uniform velocity of $20\, ms^{-1}$. The driver applies the brakes and the car comes to rest in 10 second. Calculate the retardation.

$$a = \frac{v - v_0}{t} = \frac{0 - 20}{10} = -2\, ms^{-2}$$

Retardation is $2\, ms^{-2}$

Graphical representation of motion

Know the term

What is a graph?

A diagrammatical representation of variation of one quantity with respect to another quantity is called a graph.

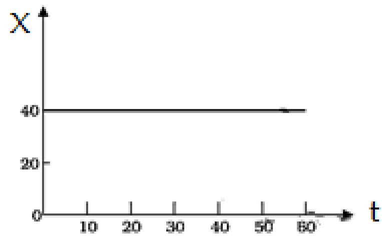
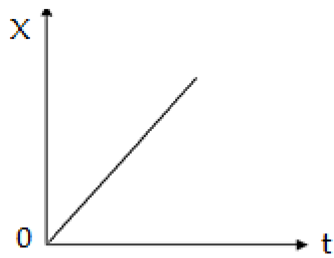
What is position-time graph?

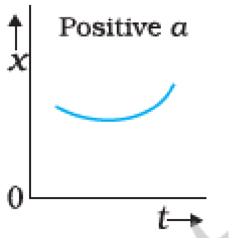
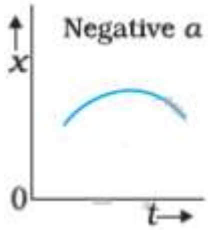
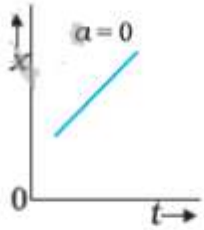
It is a graph obtained by plotting instantaneous positions of a particle versus time.

What is the significance of position- time graph?

The slope of the position time graph gives the velocity of the particle.

Represent the some types of motion on position-time graphs.

Slno	Type of motion	Position-time graph
1	Object at rest	
2	Uniform motion along a straight line	

3	Motion with positive acceleration	
4	Motion with negative acceleration	
5	Motion with zero acceleration	

What is velocity time-graph?

A graph of velocity versus time is called velocity-time graph.

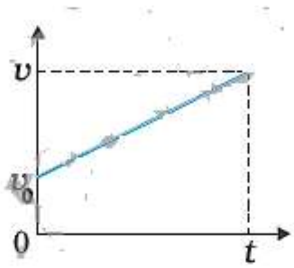
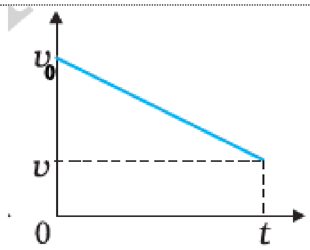
Write the significance of velocity-time (v-t) graph

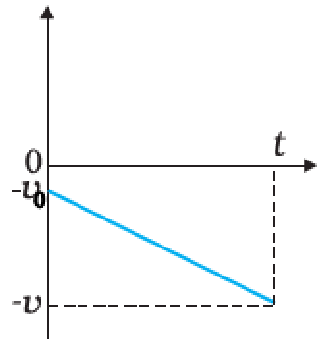
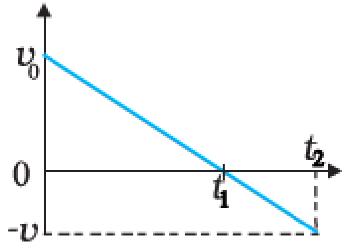
* The area under v-t graph with time axis gives the value of displacement covered in given time.
* The slope of tangent drawn on graph gives instantaneous acceleration.

What are the uses of velocity-time (v-t) graph?

* It is used to study the nature of the motion.
* It is used to find the velocity of the particle at any instant of time.
* It is used to derive the equations of motion.
* It is used to find displacement and acceleration.

Represent some types of motion on velocity time-graphs

Slno	Type of motion	v-t graph
1	Motion in positive direction with positive acceleration or uniform acceleration having some initial velocity.	
2	Motion in positive direction with negative acceleration having some initial velocity.	

3	Motion in negative direction with negative acceleration having some initial velocity.	
4	Motion of an object with negative acceleration that changes direction at time t_1 having some initial velocity.	

Kinematic equations for uniformly accelerated motion

What are kinematic equations for uniformly accelerated motion?

For uniformly accelerated motion, we can derive some simple equations that relate displacement (x), time taken (t), initial velocity (v_0), final velocity (v), and acceleration (a). These equations are called Kinematic equations for uniformly accelerated motion.

Which are the kinematic equations for uniformly accelerated motion?

The equations are, (i) $v = v_0 + at$

(ii) $x = v_0 t + \frac{1}{2}at^2$

(iii) $v^2 = {v_0}^2 + 2ax$

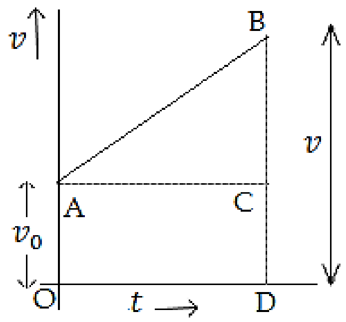
Derive the equations of motion by graphical method

(i) $\boldsymbol{v = v_0 + at}$

Consider a particle in motion with initial velocity v_0 and constant acceleration a.

Let v be the final velocity of the body at time t.

From graph,



$$\text{Slope} = \frac{BC}{AC} = \frac{BD - CD}{AC}$$

But, $CD = OA$ and $AC = OD$

$$\text{Slope} = \frac{BD - OA}{OD} = \frac{v - v_0}{t}$$

But, slope of v-t graph gives the acceleration.

$$a = \frac{v - v_0}{t}$$

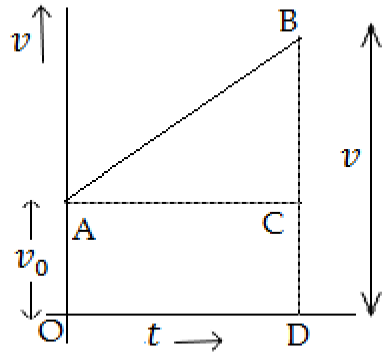
$at = v - v_0$

$v - v_0 = at$

$\boldsymbol{v = v_0 + at}$

(ii) $x = v_0 t + \frac{1}{2}at^2$

Consider a particle in motion with initial velocity v_0 and constant acceleration a.
Let v be the final velocity of the body at time t.
From graph,



$Displacement = Area\ under\ v - t\ graph$

$x = Area\ of\ trapezium\ OABD$

$$x = \frac{1}{2}(OA + BD)AC$$

$$x = \frac{1}{2}(v_0 + v)t$$

But, $v = v_0 + at$

$$x = \frac{1}{2}(v_0 + v_0 + at)t$$

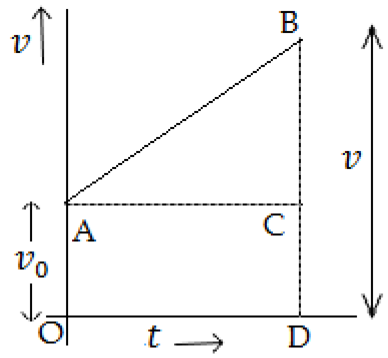
$$x = \frac{1}{2}(2\,v_0 + at)t$$

$$x = \frac{1}{2} \times 2\,v_0 \times t + \frac{1}{2} \times at \times t$$

$$\boldsymbol{x = v_0 t + \frac{1}{2}at^2}$$

(iii) $v^2 = v_0^2 + 2ax$

Consider a particle in motion with initial velocity v_0 and constant acceleration a.
Let v be the final velocity of the body at time t.
From graph,



$Displacement = Area\ under\ v - t\ graph$

$x = Area\ of\ trapezium\ OABD$

$$x = \frac{1}{2}(OA + BD)AC$$

$$x = \frac{1}{2}(v_0 + v)t$$

But, $v = v_0 + at$ and $t = \dfrac{v - v_0}{a}$

$$x = \frac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right)$$

$$x = \frac{1}{2}\frac{(v^2 - v_0^2)}{a}$$

$$2ax = v^2 - v_0^2$$

$$v^2 - v_0^2 = 2ax$$

$$\boldsymbol{v^2 = v_0^2 + 2ax}$$

Keep in mind

The set of above equations were obtained by assuming that at $t = 0$, the position of the Particle x is 0 (zero). When at $t = 0$, If the position of the particle is at $x_0(non\ zero)$, then the equations are,

(i) $v = v_0 + at$ (ii) $x - x_0 = v_0 t + \frac{1}{2}at^2$ (iii) $v^2 = {v_0}^2 + 2a(x - x_0)$

What is free fall?

An object released near the surface of the earth is accelerated downward under the influence of the force of gravity. If the air resistance is neglected, then the motion of the body is known as free fall.

What is acceleration due to gravity?

Acceleration produced in object due to gravity is called acceleration due to gravity, denoted by g.

Keep in mind

Free fall is an example for motion along a straight line under constant acceleration.

* Acceleration due to gravity is always a downward vector directed towards the centre of the earth.
* The magnitude of g is approximately $9 \cdot 8ms^2$ near the surface of the earth.
* Acceleration due to gravity is the same for all freely falling bodies irrespective of their size, shape and mass.

State the Galileo's law of odd numbers.

The distance traversed by a body falling freely from rest during equal intervals of time are in the ratio 1: 3: 5: 7: ... ... this is known as Galileo's law of ODD numbers.

Write the equations of motion under gravity?

The motion of a freely falling body is in Y-direction. If we take vertically upward as positive Y-axis, acceleration is along the negative Y-axis, therefore $a = -g$. Then,

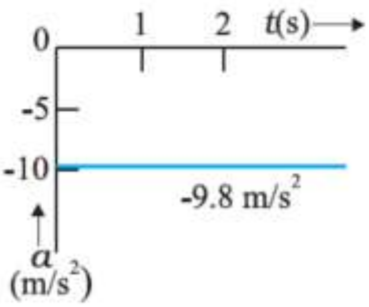
(i) $v = v_0 - gt$ $\quad$ $y = v_0 t - \frac{1}{2}gt^2$ $\quad$ $v^2 = {v_0}^2 - 2gy$

Write the equations motion for freely falling body.

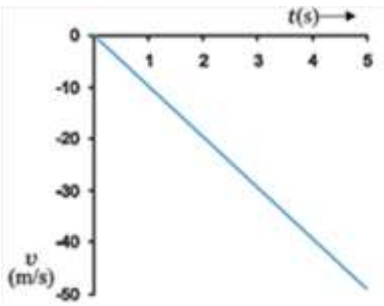
For a freely falling body the initial velocity, $v_0 = 0$. Then,

(i) $v = -gt$ $\quad$ (ii) $y = -\frac{1}{2}gt^2$ $\quad$ (iii) $v^2 = -2gy$

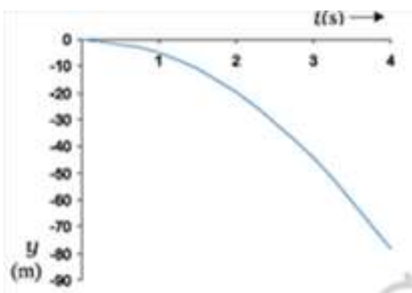
Represent the motion of a body released from rest on $a - t$ graph, $v - t$ graph and $y - t$ graph respectively.



$a - t$ graph



$v - t$ graph



$y - t$ graph

Keep in mind

What is stopping distance?

When breaks are applies to a moving vehicle, the distance travelled before stopping is called stopping distance.

$$\boldsymbol{d_s = \frac{-v_0^2}{2a}}$$

It is an important factor for road safety and it depends on initial velocity and deceleration $(-a)$.

What is reaction time?

When a situation demands our immediate action, it takes some time before we really respond this time is called reaction time.

Chapter 3 MOTION IN A PLANE

Scalar and vector quantities

What is a scalar quantity? Give examples.
A physical quantity having only magnitude is called a scalar quantity.
Ex: mass, length, temperature, speed, charge, area etc.

What is a vector quantity? Give examples.
A Physical quantity having both magnitude and direction and obey the triangle law of addition is called Vector quantity.
Ex: Displacement, velocity, acceleration, force, momentum etc.

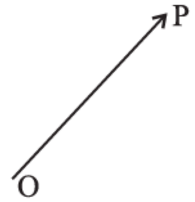
Keep in mind

* Scalar quantities are specified completely by a single number along with proper unit.
* Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers. They follow the rules of algebra.
* Vector quantities are represented by a number with an appropriate unit and direction.

What are the differences between Scalar quantity and Vector quantity? OR Distinguish between scalar and vectors.

Scalar Quantity	Vector Quantity
It has only magnitude	It has both magnitude and direction
They follow the rules of ordinary algebra	They follow the rules of vector algebra
Ex: Mass, Length, Temperature, Area	Ex: Displacement, velocity, Acceleration, Force
These change when magnitude changes	These changes when magnitude changes or direction changes or both of them changes.

How do you represent a vector diagrammatically?
Here O is called the initial point and P is called the terminal point.
Length of the line segment OP represents the magnitude and arrow at the end point indicates the direction.

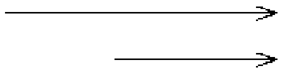


How do you represent a vector mathematically?
To represent a vector we use a bold face letters or an arrow placed over a letter.
Ex: $\mathbf{a} = \vec{a} = \overrightarrow{OP}$

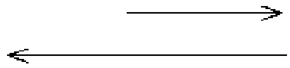
What is absolute value of a vector?
The magnitude of a vector is often called the absolute value and indicated by, $|\mathbf{a}| = |\vec{a}| = a$

Explain the classification of vectors.
(a) Parallel vectors: Two or more vectors having same direction are called parallel vectors.

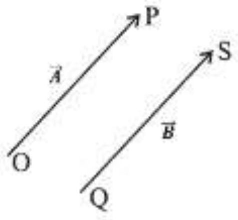
(b) Anti-parallel vectors (opposite vectors) : Vectors having opposite directions are called anti-parallel vectors (opposite vectors).



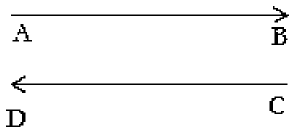
Parallel vectors



Anti-Parallel vectors



(c) Equality of vector (Equal vectors): Two (or more) vectors having same magnitude and direction, representing the same physical quantity are called Equal vectors.



$\overrightarrow{AB}$ is given vector and
$\overrightarrow{CD}$ is negative vector of $\overrightarrow{AB}$

(d) Negative of a vector: A vector having same magnitude but having opposite direction to that of the given vector is called negative of a given vector.

(e) Zero(Null) vector: A vector whose magnitude is zero is called Zero vector. It is represented by $\vec{0}$ and

(f) Unit vector: A vector having unit magnitude is called unit vector **<u>OR</u>** a vector whose magnitude is equal to one is called unit vector.

(g) Concurrent vectors: The vectors having same initial point are called concurrent vectors.

(h) Co-planar vectors: The vectors acting in the same plane are called coplanar vectors.

What is the direction of a zero vector?
The direction of zero vector is not specified.

List the properties of a zero vector.
(i) $\vec{A} - \vec{A} = \vec{0}$
(ii) $|\vec{0}| = 0$
(iii) $\vec{A} - \vec{0} = \vec{A}$
(iv) $\vec{0}\,\vec{A} = \vec{A}\,\vec{0} = \vec{0}$
(v) $\lambda\,\vec{0} = \vec{0}$

Represent a unit vector mathematically.

mathematically unit vector can be represented as, $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

What is the purpose of a unit vector?
Its purpose is to specify a direction. Unit vector has no dimensions and unit.

Keep in mind

* If $\vec{a}$ is a vector, then the unit vector in direction of $\vec{a}$ is written as $\hat{a}$ (read as "a cap") $\vec{a} = |\vec{a}|\,\hat{a}$,
* The unit vectors in the positive directions of x, y and z axes are labelled as $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ respectively.

What is resultant vector?

The resultant vector is a single vector whose effect is the same as the effect produced by the individual vectors together.

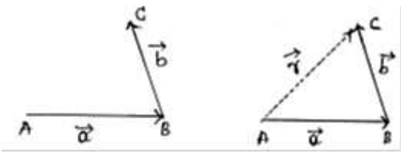
Can we add or subtract two vectors?

Yes, two vectors representing the same quantity in the same unit can be added or subtracted.

State and explain law of triangle of vectors or triangular law of vector addition.

If two vectors $\vec{a}$ and $\vec{b}$ are represented by two sides of a triangle in head to tail form, then the closing side of the triangle taken from tail of the first to head of the second represent the vector sum of $\vec{a}$ and $\vec{b}$.

Explanation:



Consider two vectors, $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{BC}$ are of same nature.

According to triangle law of addition, $\overrightarrow{AC} = \vec{r}$ represents the sum of $\vec{a}$ and $\vec{b}$.

$\vec{a} + \vec{b} = \vec{r}$ or $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Mention the properties vector addition:

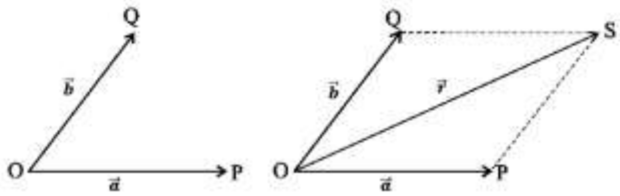
(a) Vector addition is commutative. $\vec{a} + \vec{b} = \vec{r} = \vec{b} + \vec{a}$

(b) Vector addition is Associative. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

State and explain law of parallelogram of vectors or parallelogram law of vector addition.

If two vectors are represented by two adjacent sides of a parallelogram, then the diagonal drawn from the common initial point represents their vector sum.

Explanation: Vector $\vec{a}$ and $\vec{b}$ are drawn with a common initial point and parallelogram is constructed using these two vectors as two adjacent sides of a parallelogram. The diagonal originating from the common initial point is vector sum of $\vec{a}$ and $\vec{b}$.

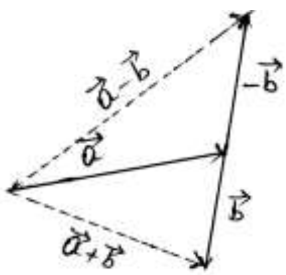


Explain the subtraction vectors in graphical method.

Subtraction of vectors can be defined in terms of addition of vectors.

Consider two vectors $\vec{a}$ and $\vec{b}$ of same nature and another vector $-\vec{b}$ which is opposite (negative) vector of $\vec{b}$, then

$$\vec{a} + (-\vec{b}) = \vec{a} + \vec{b}$$



Keep in mind

* In triangular method of vector addition, vectors are arranged head-to-tail. Hence it is called head-to-tail method.
* Subtraction vector is neither commutative nor associative.

Can we multiply a vector by a scalar? Give examples.

Yes, we can multiply a vector by a scalar irrespective of their nature and dimensions.

Ex: $\vec{F} = m\vec{a}, \vec{p} = m\vec{v}$ and $\vec{J} = \vec{F}t$

Explain the multiplication of a vector by real(Scalar) number OR Scalar multiplication of a Vector.

The product of a vector $\vec{v}$ and a positive number (Scalar) λ gives a vector, whose magnitude is changed by a factor λ but direction is same as that of $\vec{v}$.

$$|\lambda\vec{v}| = \lambda|\vec{v}| \quad (if\ \lambda > 0)$$

If λ is negative, the direction of the vector $\lambda\vec{v}$ is opposite to the direction of the vector $\vec{v}$ and magnitude is $-\lambda$ times $|\vec{v}|$.

What happens to the dimensions of vector quantity when we multiplied by a scalar quantity?

(i) If the multiplying factor is dimensionless then, the product have the same dimensions as that of given vector.

(i) If the multiplying factor has dimensions then, the product have the product of dimensions of given vector and multiplying factor.

What are components of a vector?

Effects of a vector in different directions are called components of a vector.

What is resolution of vectors?

Splitting a given vector into a number of components is called resolution of vectors **OR**

The process of finding the components of a given vector is called resolution the vector.

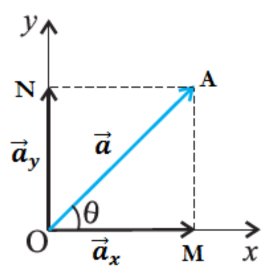
What is meant by rectangular components?

If the components of a vector are perpendicular to each other, then the components are called rectangular components.

Obtain the expressions for X and Y components (Rectangular components) of a Vector:

Consider a vector $\overrightarrow{OA} = \vec{a}$ in X-Y plane, which makes an angle θ with the positive X-axis.

Draw AM and AN perpendicular to X and Y axes respectively.



Let $\overrightarrow{OM} = \vec{a}_x$ and $\overrightarrow{ON} = \vec{a}_y$

From parallelogram law of addition, we have $\overrightarrow{OM} + \overrightarrow{ON} = \overrightarrow{OA}$

$$\vec{a}_x + \vec{a}_y = \vec{a}$$

(Here $\vec{a}_x$ is x - component of $\vec{a}$ and $\vec{a}_y$ is y - component of $\vec{a}$)

From $\Delta^{le}\ OAM$, $\quad cos\,\theta = \frac{OM}{OA} = \frac{a_x}{a}$

$$\boldsymbol{a_x = a\ \cos\theta}$$

and $\quad sin\,\theta = \frac{AM}{OA} = \frac{a_y}{a}$

$$\boldsymbol{a_y = a\ \sin\theta}$$

Vectoricaly $\quad \boldsymbol{\vec{a}_x = \vec{a}\ \cos\theta}$ and $\boldsymbol{\vec{a}_y = \vec{a}\ \sin\theta}$

Illustrations.

A vector of 10 unit acts at a point making an angle 30^0 with the horizontal. What are the horizontal and vertical components of the vector?

We have, $\vec{a} = \vec{a}_x + \vec{a}_y = \vec{a}\ \cos\theta + \vec{a}\ \sin\theta$

$a_x = a\ \cos\theta = 10 \times \cos 30^0 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\, units$

$a_y = a\ \sin\theta = 10 \times \sin 30^0 = 10 \times \frac{1}{2} = 5\, unit$

Obtain the expressions for magnitude and direction of vector in terms of their rectangular components.

Magnitude: Magnitude of $\vec{a}$ is given by $|\vec{a}| = a$

Now, $a_x^2 = a^2\cos^2\theta$ and $a_y^2 = a^2\sin^2\theta$

Taking $a_x^2 + a_y^2 = a^2\cos^2\theta + a^2\sin^2\theta$

$$a_x^2 + a_y^2 = a^2(\cos^2\theta + \sin^2\theta)$$

$$a_x^2 + a_y^2 = a^2$$

$$\boldsymbol{a = \sqrt{a_x^2 + a_y^2}}$$

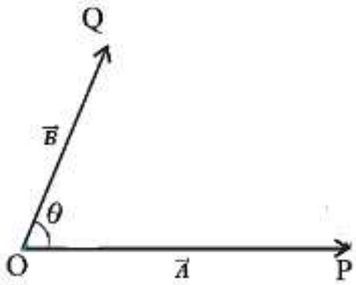
Direction: By taking $\frac{a_y}{a_x} = \frac{a\ \sin\theta}{a\ \cos\theta}$

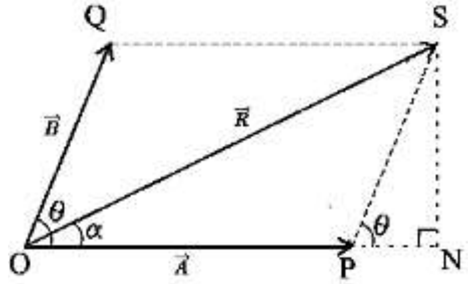
$$\frac{a_y}{a_x} = \tan\theta$$

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

Find the magnitude and direction of the resultatnt of two vectors $\vec{A}$ and $\vec{B}$ in terms of their magnitudes and angle θ between them. <u>**OR**</u>

Derive an expression for magnitude and direction of resultant of two concurrent vectors.





Let $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ represent the two vectors $\vec{A}$ and $\vec{B}$ making an angle θ.

Then using the parallelogram method of vector addition $\overrightarrow{OS}$ represents the resultant vector $\vec{R}$.

$$\vec{R} = \vec{A} + \vec{B}$$

Draw SN is normal to OP extended.

In $\Delta^{le}\ SPN$, $\cos\theta = \frac{PN}{SP}$

$PN = SP\cos\theta = B\cos\theta$

and $\sin\theta = \frac{SN}{SP}$

$SN = SP\sin\theta = B\sin\theta$

Magnitude: From geometry,

$$OS^2 = ON^2 + SN^2$$

$$OS^2 = (OP + PN)^2 + SN^2$$

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + (B\cos\theta)^2 + 2AB\cos\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\boldsymbol{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$

Direction: Let α be angle made by the resultant vector $\vec{R}$ with the vector $\vec{A}$, then

$$\tan\alpha = \frac{SN}{ON} = \frac{SN}{OP + PN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\boldsymbol{\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)}$$

Keep in mind

* The magnitude of the resultant of two vectors is maximum when angle between them is 0^0.
* The magnitude of the resultant of two vectors is minimum when angle between them is 180^0.

Illustrations.

If $\vec{A} = \vec{B}$ and are acting at right angles to each other, what is the magnitude of their resultant?

$\vec{R} = \sqrt{A^2 + B^2} = \sqrt{A^2 + A^2} = \sqrt{2A^2} = \sqrt{2}\,A$

$\vec{A} = 3\ units$ acting along east, and $\vec{B} = 4\ units$ acting along north. What is the magnitude of their resultant?

$\vec{R} = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5\ units$

What are the limitations of Graphical method of adding vectors?

(i) It is very difficult method.

(ii) It has limited accuracy.

To overcome these limitations Analytical method of addition of vectors is preferred.

Explain the addition of vectors by analytical method.

In two Dimensions: Consider two vectors $\vec{a}$ and $\vec{b}$ in X-Y plane.

If $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ and $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath}$ then, $\vec{R} = \vec{a} + \vec{b}$

$$\vec{R} = (a_x\hat{\imath} + a_y\hat{\jmath}) + (b_x\hat{\imath} + b_y\hat{\jmath})$$

$$\vec{R} = a_x\hat{\imath} + b_x\hat{\imath} + a_y\hat{\jmath} + b_y\hat{\jmath}$$

$$\vec{R} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath}$$

$\boldsymbol{\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}}$ Where $R_x = a_x + b_x$ and $R_y = a_y + b_y$

In three Dimensions: If $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ and $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$ then, $\vec{R} = \vec{a} + \vec{b}$

$$\vec{R} = (a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}) + (b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k})$$

$$\vec{R} = a_x\hat{\imath} + b_x\hat{\imath} + a_y\hat{\jmath} + b_y\hat{\jmath} + a_z\hat{k} + b_z\hat{k}$$

$$\vec{R} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}$$

$$\boldsymbol{\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}}$$

This method can be extended to addition and subtraction of any number of vectors.

Illustrations.

What is the magnitude of $\vec{A} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$$\vec{A} = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9+4+1} = \sqrt{14}$$

What is the unit vector of $\vec{P} = 3\hat{\imath} - 4\hat{\jmath}$

$$\hat{P} = \frac{\vec{P}}{|\vec{P}|} = \frac{3\hat{\imath} - 4\hat{\jmath}}{\sqrt{3^2 + 4^2}} = \frac{3\hat{\imath} - 4\hat{\jmath}}{5} = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath}$$

Find the sum of $\vec{A} = 5\hat{\imath} - 2\hat{\jmath} + 9\hat{k}$ and $\vec{B} = 3\hat{\imath} + 7\hat{\jmath}$

$$\vec{A} + \vec{B} = \vec{R} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}$$

$$\vec{R} = (5+3)\hat{\imath} + (-2+7)\hat{\jmath} + (9+0)\hat{k} = 8\hat{\imath} + 5\hat{\jmath} + 9\hat{k}$$

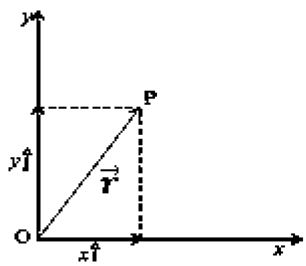
Motion in a plane

What is position vector?

A vector which gives the position of a particle with reference to the origin of a co-ordinate system is called position vector.

The position vector $\vec{r}$ of a particle is given by, $\boldsymbol{\vec{r} = x\hat{\imath} + y\hat{\jmath}}$

where x and y are component of $\vec{r}$ along X-axis and Y-axis respectively.



Represent displacement of a particle in two dimensions.

Consider a particle moves along curve.

Initially it is at P_1 at time t_1 and moves to a new position P_2 at time t_2.

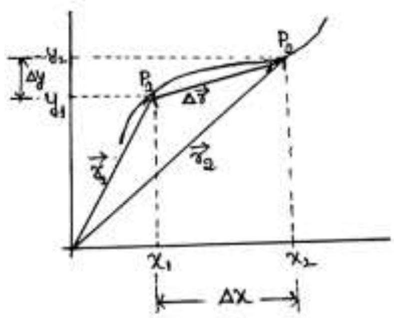
Then the displacement is given by, $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta\vec{r} = (x_2\hat{\imath} + y_2\hat{\jmath}) - (x_1\hat{\imath} + y_1\hat{\jmath})$$

$$\Delta\vec{r} = (x_2\hat{\imath} - x_1\hat{\imath}) + (y_2\hat{\jmath} - y_1\hat{\jmath})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$

$\boldsymbol{\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}}$ Where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$



Represent average velocity in two dimensions.

Average velocity: It is defined as ratio of the displacement to the time taken.

$$\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{\imath} + \Delta y\hat{\jmath}}{\Delta t}$$

$$\vec{\bar{v}} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$

$$\boldsymbol{\vec{\bar{v}} = \bar{v}_x\hat{\imath} + \bar{v}_y\hat{\jmath}} \qquad \text{Where } \bar{v}_x = \frac{\Delta x}{\Delta t} \text{ and } \bar{v}_y = \frac{\Delta y}{\Delta t}$$

Direction of the average velocity is same as that of the displacement.

Represent instantaneous velocity (Velocity) in two dimensions.

Instantaneous velocity: It is given by the limiting value of the average velocity as the time interval approaches to zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\boldsymbol{\vec{v} = \frac{d\vec{r}}{dt}}$$

The direction of velocity at any point on the path of the object is tangential to the path at that point and in the direction of the motion.

The components of the velocity $\vec{v}$ are given by,

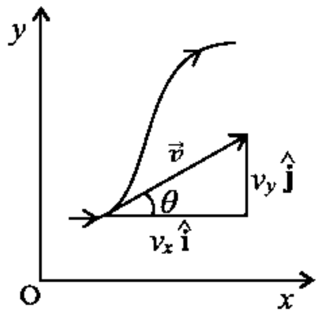
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \hat{\imath} + \frac{\Delta y}{\Delta t} \hat{\jmath} \right)$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \hat{\jmath}$$

$$\vec{v} = \frac{dx}{dt} \hat{\imath} + \frac{dy}{dt} \hat{\jmath}$$

$$\boldsymbol{\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath}}$$



The magnitude is given by $\boldsymbol{v = \sqrt{v_x^2 + v_y^2}}$ and Direction is given by $\boldsymbol{\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$

Represent average acceleration in two dimensions.

Average acceleration: It is defined as the change in velocity divided by time interval.

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta\left(v_x \hat{\imath} + v_y \hat{\jmath}\right)}{\Delta t}$$

$$\vec{\bar{a}} = \frac{\Delta v_x}{\Delta t} \hat{\imath} + \frac{\Delta v_y}{\Delta t} \hat{\jmath}$$

$$\boldsymbol{\vec{\bar{a}} = \bar{a}_x \hat{\imath} + \bar{a}_y \hat{\jmath}}$$

Represent instantaneous acceleration (acceleration) in two dimensions.

Instantaneous acceleration: It is the limiting value of the average acceleration as the time interval approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\boldsymbol{\vec{a} = \frac{d\vec{v}}{dt}}$$

The Components are given by,

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta(v_x \hat{\imath} + v_y \hat{\jmath})}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t} \hat{\jmath}$$

$$\vec{a} = \frac{dv_x}{dt} \hat{\imath} + \frac{dv_y}{dt} \hat{\jmath}$$

$$\boldsymbol{\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath}}$$

Keep in mind

* In one dimension the direction of velocity and acceleration is same or in opposite direction but in two or three dimensions, velocity and acceleration vectors may have any angle between 0^0 and 180^0.

Write the kinematic equations of motion in a plane with constant acceleration.

Consider an object moving in x-y plane and its acceleration $\vec{a}$ is constant.

Let the velocity of the object be $\vec{v}_0$ at time $t = 0$ and $\vec{v}$ at time t, then

(i) $\vec{v} = \vec{v}_0 + \vec{a}t$

In terms of its components, $v_x = v_{0x} + a_x t$

$v_y = v_{0y} + a_y t$

(ii) Displacement is $\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

In terms of its components, $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$

$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$

(iii) $\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{r} - \vec{r}_0)$

In terms of its components, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

The motion in plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

What is two-dimensional motion?

Motion of the particle in a plane is called two-dimensional motion.

(i) Projectile motion and (ii) Circular motion are two-dimensional motions.

Why two-dimensional motion in a curve is an accelerated motion?

When a particle traces a curve in two dimensional plane, the velocity of the particle changes at least in direction. Hence, a two dimensional motion along a curve is essentially an accelerated motion. Acceleration may be uniform or non-uniform.

Projectile motion

What is a projectile? Give examples.

A projectile is any object thrown into air or space.

Ex: A ball leaving the hand of a bowler, a stone thrown at an angle to the horizontal, an object dropped from an aeroplane in horizontal flight.

What is projectile motion?

Motion associated with a projectile in parabolic path is called Projectile motion.

Keep in mind

* The motion of projectile may be thought of as the result of two separate, simultaneously occurring components of motion. One component is *along a horizontal direction without any acceleration* and other *along the vertical direction with constant acceleration due to gravity*.
* *It was Galileo, who first stated this independency of the horizontal and vertical components of projectile motion.*

What is projectile velocity?

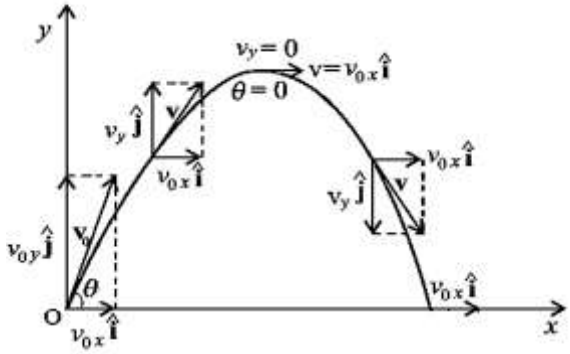
It is the velocity with which the projectile is projected.

What is angle of projection?

It is the angle made by the projectile with the horizontal

Analyse the projectile motion and represent it graphically.

Let a projectile is projected with initial velocity $\vec{v}_0$ that makes an angle θ with x-axis.



The acceleration acting on it is due to gravity and is directed vertically downwards.

$a_x = 0$, $\quad a_y = -g$, hence $\vec{a} = -g\hat{j}$

The components of initial velocity $\vec{v}_0$ are,

$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

The components of velocity at time t are,

$$v_x = v_0 \cos\theta$$
$$v_y = v_0 \sin\theta - gt$$

The components of displacements at time t are,

(i) $x = v_{0x}t + \frac{1}{2}a_x t^2$

$x = v_{0x}t \qquad (\because a_x = 0)$

$\boldsymbol{x = (v_0 \cos\theta)\, t} \qquad$ (along X-axis)

(ii) $y = v_{0y}t + \frac{1}{2}a_y t^2$

$\boldsymbol{y = (v_0 \sin\theta)t - \frac{1}{2}gt^2} \qquad$ (along $Y-$ axis)

What is trajectory of a projectile?

The path described by the projectile is called trajectory.

What is the nature of trajectory (path) of a projectile?

The trajectory (path) is a parabola.

Derive the expression for trajectory (path) of a projectile <u>OR</u>
Show that the path of a projectile is Parabola.

The displacement of the projectile along X-axis is,

$$x = (v_0 \cos\theta)\, t$$
$$t = \frac{x}{v_0 \cos\theta}$$

The displacement of the projectile along Y-axis is,

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$
$$y = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2$$
$$y = \frac{\sin\theta}{\cos\theta}\, x - \frac{1}{2}\, \frac{g}{v_0^2 \cos^2\theta}\, x^2$$
$$y = \tan\theta\; x - \frac{1}{2}\, \frac{g}{v_0^2 \cos^2\theta}\, x^2$$

$\boldsymbol{y = ax - bx^2} \qquad$ where $a = tan\,\theta$ and $b = \frac{1}{2}\frac{g}{v_0^2\, cos^2\,\theta}$

The equation $y = ax - bx^2$ represents a ***parabola.*** Hence the trajectory is a parabola.

What is time of Flight of a projectile?

It is the time during which the projectile is in flight. It is denoted by T_f.

Derive the expression for Time of flight of a projectile.

The component of velocity along Y-axis at time t is,

$$v_y = v_0 \sin\theta - gt$$

At maximum height $v_y = 0$ and time for maximum height, $t = t_m$.

$$0 = v_0 \sin\theta - gt_m$$

$$gt_m = v_0 \sin\theta$$

$$t_m = \frac{v_0 \sin\theta}{g}$$

Time of flight $T_f = 2t_m$ because "time of ascent = time of descent"

$$\boldsymbol{T_f = \frac{2v_0 \sin\theta}{g}}$$

What is maximum height of a projectile?

It is the maximum vertical distance travelled by the projectile in time t_m. It is denoted by h_m.

Derive the expression for maximum height of a projectile.

The displacement along Y – axis is, $y = (v_0 \sin\theta)t_m - \frac{1}{2}gt_m^2$

$$h_m = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$h_m = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2}g\,\frac{(v_0 \sin\theta)^2}{g^2}$$

$$h_m = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2}\,\frac{(v_0 \sin\theta)^2}{g}$$

$$h_m = \left(1 - \frac{1}{2}\right)\frac{(v_0 \sin\theta)^2}{g}$$

$$h_m = \frac{1}{2}\,\frac{(v_0 \sin\theta)^2}{g}$$

$$\boldsymbol{h_m = \frac{(v_0 \sin\theta)^2}{2g}}$$

What is horizontal Range of projectile?

It is the horizontal distance covered by the projectile during its flight. It is denoted by R.

Derive the expression for Horizontal Range of projectile/

Displacement along X-axis is, $x = (v_0 \cos\theta)\, t$

Now $x = R$ and $t = T_f$

$$R = (v_0 \cos\theta)\, T_f$$

$$R = (v_0 \cos\theta)\left(\frac{2v_0 \sin\theta}{g}\right)$$

$$R = \frac{v_0^2}{g}\, 2\cos\theta \sin\theta$$

$$\boldsymbol{R = \frac{v_0^2}{g} \sin 2\theta}$$

At what angle of projection the range of a projectile will be maximum?

For a given speed of projection, the projectile will have *maximum* range (R_m) when *angle of projection is* 45^0.

Obtain the expression for Maximum range of a projectile.

For maximum range the angle of projection, $\theta = 45^0$

Maximum range, $R_m = \frac{v_0^2}{g}\sin 2(45^0)$

$$\boldsymbol{R_m = \frac{v_0^2}{g}}$$

Show that $R_m = 4h_m$ when the angle of projection is $\theta = 45^0$.

For $\theta = 45^0$, $R_m = \frac{v_0^2}{g}$ and

$$h_m = \frac{v_0^2 \sin^2\theta}{2g} = \frac{v_0^2 \sin^2 45}{2g} = \frac{v_0^2}{2g} \times \frac{1}{2}$$

$$h_m = \frac{v_0^2}{4g} = \frac{1}{4}\left(\frac{v_0^2}{g}\right)$$

$$h_m = \frac{1}{4}(R_m)$$

$$\boldsymbol{R_m = 4h_m}$$

∴ The maximum range of a projectile is equal to 4 times the maximum height reached.

Illustrations.

For what two angles of projection, the range of projectile is same?

θ and $90 - \theta$

Three athletes A,B and C participating in a long jump event jump by making angles 30^0, 45^0 and 60^0 with the ground respectively. Who will be the winner?

Athlete B.

Uniform circular motion

What is uniform circular motion?

Motion of the object in a circular path at a constant speed is called uniform circular motion.

Why uniform circular motion is accelerated motion?

Even though the object moves at a constant speed it has acceleration, because there is a continuous change in its direction of motion. Hence there is a change in its velocity from point to point.

What is the direction of velocity of a particle in a uniform circular motion?

Velocity is directed along the tangent to the circular path.

What is the direction of acceleration of a particle in a uniform circular motion?

Acceleration is directed towards the centre along the radius.

What is the angle between direction of velocity and direction of acceleration in uniform circular motion?

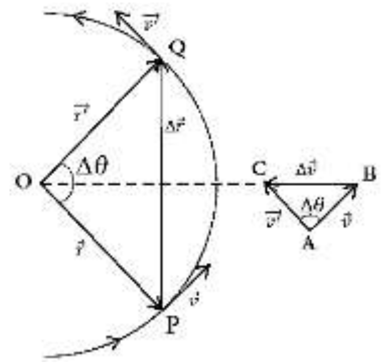
90^0 **<u>OR</u>** perpendicular to each other.

What is centripetal acceleration?

The acceleration, which is directed towards the centre, is called centripetal acceleration.

Derive the expression for centripetal acceleration.

Let $\vec{r}$ and $\vec{r'}$ be the position vectors and $\vec{v}$ and $\vec{v'}$ are the velocities of the object when it is at P and Q as shown.



The average acceleration $= \dfrac{\Delta\vec{v}}{\Delta t}$

Since $\Delta\vec{v}$ is perpendicular to $\Delta\vec{r}$, $\bar{a}$ is along $\Delta\vec{v}$ and perpendicular to $\Delta\vec{r}$ and directed towards the centre of the circle.

The Instantaneous acceleration is, $\vec{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t}$

Its magnitude is given by, $a_c = |\vec{a}|$

$$a_c = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t}$$

Since the velocity vectors $\vec{v}$ and $\vec{v'}$ are always perpendicular to $\vec{r}$ and $\vec{r'}$, the angle between $\vec{v}$ and $\vec{v'}$ is also $\Delta\theta$

$\therefore$ ΔABC and ΔOPQ are similar.

Then, $\dfrac{BC}{PQ} = \dfrac{AB}{OP}$

$$\frac{|\Delta\vec{v}|}{|\Delta\vec{r}|} = \frac{v}{R} \qquad (\because |\vec{r}| = R = radius)$$

$$\Delta\vec{v} = \frac{|\Delta\vec{r}|\, v}{R}$$

$$\therefore a_c = \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|\, v}{\Delta t\, R}$$

$$a_c = \frac{v}{R} \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t}$$

$$a_c = \frac{v}{R}\, v$$

$$\boldsymbol{a_c = \frac{v^2}{R}}$$

This equation represents the magnitude of acceleration and is directed towards centre.

Keep in mind

* The term centripetal acceleration was termed by Newton and *Centripetal* comes from a Greek term which means Centre seeking of towards centre.

What is time period of revolution?

Time taken by an object to make one revolution is called time period of revolution.

What is Frequency of revolution?

Number of revolution made in one second is known as frequency.

$$\nu = \frac{1}{T}$$

Obtain the Alternative expression for velocity and acceleration

(i) In terms of angular velocity:

Angular speed, $\omega = \dfrac{\Delta\theta}{\Delta t}$

If the distance travelled $PQ = \Delta s$ then,

Speed, $\quad v = \dfrac{\Delta s}{\Delta t}$

But $\Delta s = R\,\Delta\theta$, where R is the radius of the trajectory.

$$\therefore v = \frac{R\,\Delta\theta}{\Delta t} = R\left(\frac{\Delta\theta}{\Delta t}\right)$$

$$\boldsymbol{v = R\omega} \quad \rightarrow (1)$$

The Centripetal acceleration $a_c = \frac{v^2}{R} = \frac{R^2\omega^2}{R}$

$$\boldsymbol{a_c = R\omega^2}$$

(ii) In terms of frequency:

Distance moved in time period $T = 2\pi R$

$$\text{Speed}, v = \frac{2\pi R}{T} = 2\pi R\nu$$

$$v = R(2\pi\nu) \quad \rightarrow (2)$$

Comparing equation (1) and (2), we get

$$\boldsymbol{\omega = 2\pi\nu}$$

Then Acceleration, $a_c = (2\pi\nu)^2 R$

$$\boldsymbol{a_c = 4\pi^2\nu^2 R}$$

Chapter 4 **LAWS OF MOTION**

State Aristotle's Law.
An external force is required to keep a body in motion.

What is fallacy in the Aristotle's law?
A moving object comes to rest because; the external force of friction on the object by the floor opposes its motion. If there is no friction no force is required to keep the object in motion.

State the law of inertia.
If the net external force is zero, a body at rest continues to be at rest and a body in motion continues to be in uniform motion.

What is inertia?
The property of a body to change its state of rest or uniform motion unless some external force acts on it, is called inertia.

Mention the types of inertia.
(i) Inertia of rest: The property of a body to remain at rest.
(ii) Inertia of motion: The property of a body to oppose the change in its motion.

Keep in mind

* Aristotle's view point about the motion of the body was rejected by Galileo and gave the law of Inertia.
* Mass of a body is measure of inertia. Generally heavier body has larger inertia than a lighter body.
* Based on the Galileo's idea, the intimate relationship between force acting on a body and its motion executed by the body was first understood by Isaac Newton.

Newton's laws of motion

State Newton's first law of motion.
Everybody continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Write the Newton's first law of motion in terms of acceleration.
If the net external force on a body is zero, its acceleration is zero. Acceleration can non zero only if there is a net external force on the body.

Give examples for Newton's first law of motion.

* A passenger in a bus is pushed back when the bus suddenly starts moving. This is due to inertia of rest of the upper part of the passenger's body.
* A man jumping from moving bus falls forward **or** a person in a moving vehicle tends to fall forward when the vehicle suddenly stops. This is due to inertia of motion of the upper part of the passenger's body.

* An athlete runs some distance, before taking a long jump due to inertia of motion, since length of the jump depend upon his velocity at the instant of jump.

What is significance of Newton's first law of motion?
Newton's first law of motion gives the definition for force and reveals Inertia, a fundamental property of all matter.

What is force?
The external agency which changes or tends to change the state of rest or state of uniform motion of a body in a straight line is called force.

Illustration
A moving bicycle comes to rest after some time, if we stop pedalling it. But Newton's first law says that everybody continues to be in its state of uniform motion, unless some external force acts on it. Is it failure of Newton's law? Discuss.
If we stop paddling a bicycle which is moving at uniform speed, the bicycle does not go on moving forever. It comes to rest after some time. The moving bicycle has been compelled to change its state of uniform motion by external force of air resistance and friction. If there were no air resistance and friction, it would not stop by itself. Hence Newton's law is applicable here.

Define linear momentum.
Linear momentum of a body is defined as the product of its mass and velocity.
Linear momentum = mass × velocity
$\boldsymbol{\vec{p} = m\vec{v}}$

Mention the SI unit and dimensions of linear momentum.
SI unit is $kgms^{-1}$. Dimensions of momentum are $[MLT^{-1}]$

Keep in mind

* Force is a vector quantity.
* The concept of momentum was introduced by Newton.
* It is a measure of the ability of a body to impart motion to another.
* Linear momentum is a vector quantity.

State Newton's Second law of motion.
The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

Mathematically, $\frac{d\vec{p}}{dt} \propto \vec{F}$

Derive of $\boldsymbol{F = ma}$ using Newton's second law of motion.
Consider a body of mass m, moving with a velocity $\vec{v_1}$ and having momentum $\vec{p_1}$.
Let a force $\vec{F}$ acts on it for time Δt.
Then velocity changes to $\vec{v_2}$ and momentum to $\vec{p_2}$.
Then change momentum is, $\vec{p_2} - \vec{p_1} = m\vec{v_2} - m\vec{v_1}$
$d\vec{p} = m(\vec{v_2} - \vec{v_1})$
$d\vec{p} = m\, d\vec{v}$

From second law of motion, $\vec{F} \propto \frac{d\vec{p}}{dt}$

$\vec{F} = k\,\frac{d\vec{p}}{dt}$

$\vec{F} = k\,\frac{m\,d\vec{v}}{dt}$

$\vec{F} = k\,m\,\frac{d\vec{v}}{dt}$

$\vec{F} = km\vec{a}$

For simplicity, we choose $k = 1$

Then, $\boldsymbol{\vec{F} = m\vec{a}}$

What is the significance of Newton's second law of motion?

It signifies momentum and gives a formula to measure the force.

Write the SI unit of Force and its dimensions.

SI unit of force is $newton$ or N and dimensions are $[MLT^{-2}]$

Define newton (N).

One newton is that force which causes an acceleration of $1ms^{-2}$ to a body of mass $1kg$.

Mention the applications of Newton's second law.

* A cricket player lowers his hands while catching a ball. Because the player increases the time during which the high velocity of the moving ball reduces to zero.
* The vehicles are fitted with shockers (springs). It is used to decrease the rate of change of momentum by increasing the time interval.
* Glass wares and china wares are wrapped with straw pieces before transportation. Soft materials like paper or straw pieces slow down the rate of change of momentum.
* A person falling on a cemented floor gets injured but a person falling on heap of sand is not.

What is impulsive force? Give examples.

Large force acting on a body for a short time is called an impulsive force.

Ex: A ball hit by bat, kicking a football, hammering a nail etc.

Define impulse.

It is the product of the force and time interval for which the force acts. It is denoted by $\vec{J}$.

$impulse(\vec{J}) = Force \times time = \vec{F}t$

Mention the SI unit of impulse and its dimensions.

SI unit of Impulse is $newton - second(Ns)$. Dimensions are $[MLT^{-1}]$

State and prove impulse-momentum theorem.

Impulse is equal to change in momentum.

Proof: Impulse, $\vec{J} = \vec{F}t$

But we have $\vec{F} = m\vec{a}$

$\vec{J} = m\vec{a}t$

$\vec{J} = m(\vec{v} - \vec{v}_0)$ $\qquad (\because v - v_0 = at)$

$\vec{J} = m\vec{v} - m\overrightarrow{v_0}$

$\vec{J} = final\ momentum - initial\ momentum$

$\vec{J} = \textbf{Change in momentum}$

Illustration

Compare the linear momenta of two bodies one of mass 5 g moving with a speed of 50 ms^{-1} and another body of mass $0.5\ kg$ moving with a speed of $0.5\ ms^{-1}$.

$p_1 = m_1v_1 = 5 \times 10^{-3} \times 50 = 250 \times 10^{-3} = 0.25\ kgms^{-1}$

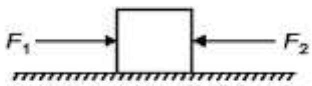
$p_2 = m_2v_2 = 0.5 \times 0.5 = 0.25 = 0.25\ kgms^{-1}$

$p_1 = p_2$

What is the change in momentum of a particle in uniform circular motion at diametrically opposite points?

$p + (-p) = 2p$

Two forces $\vec{F}_1 = 10\ N$ and $\vec{F}_2 = 2\ N$acting on a body of mass 2 kg as shown. Calculate the acceleration produced.



Resultant force, $\vec{F} = \vec{F}_1 - \vec{F}_2 = 10 - 2 = 8\ N$

Acceleration, $a = {}^F/_m = {}^8/_2 = 4\ ms^{-2}$

The rate of change of momentum of a body is 5 $kgms^{-1}$. What is the force acting on the body?

$$\vec{F} = \frac{d\vec{p}}{dt} = 5\ N$$

State Newton's third law of motion.

To every action there is always an equal and opposite reaction.

Mathematically, $Force\ on\ A\ by\ B = -(Force\ on\ B\ by\ A)$

$$\vec{\boldsymbol{F}}_{AB} = -\vec{\boldsymbol{F}}_{BA}$$

What is the significance of Newton's third law?

It signifies that forces never occur singly in nature, but they always occur in pairs.

Which law is used in launching of rocket?

Launching of rocket is based on this law.

Illustrate Newton's third law of motion with examples.

* When a person jumps from a boat, he pushes the boat in the backward direction while the boat pushes him in the forward direction.
* A swimmer pushes the water in the backward direction and the water pushes the swimmer in the forward direction.
* A person walking on the floor. When we walk on the ground our foot pushes the ground backward and in turn the ground pushes our foot forward.

Keep in mind

* Impulse and momentum have same dimensions.
* Impulse is vector quantity.
* The term action and reaction means the force.

* Action and reaction forces act at the same instant on different bodies, not on the same body. So they do not cancel each other.

State the law of conservation of linear momentum.
The total momentum of an isolated system of interacting particle is conserved.

Which principle is used in motion of rocket?
The motion of rocket is based on this principle.

Prove law of conservation of linear momentum using Newton's laws of motion.
Consider two bodies A and B, with initial momentum $\vec{p}_A$ and $\vec{p}_B$.
Let the bodies collide and get apart with final momentum $\overrightarrow{p_A'}$ and $\overrightarrow{p_B'}$ respectively.
From Newton's second law,

Force on A by B is, $\vec{F}_{AB} = \dfrac{\overrightarrow{p_A'} - \vec{p}_A}{dt}$ and

Force on B by A is, $\vec{F}_{BA} = \dfrac{\overrightarrow{p_B'} - \vec{p}_B}{dt}$ where dt is time for which the bodies are in contact.

But from Newton's third law, $\vec{F}_{AB} = -\vec{F}_{BA}$

$$\frac{\overrightarrow{p_A'} - \vec{p}_A}{dt} = -\left(\frac{\overrightarrow{p_B'} - \vec{p}_B}{dt}\right)$$

$$\overrightarrow{p_A'} - \vec{p}_A = -\left(\overrightarrow{p_B'} - \vec{p}_B\right)$$

$$\overrightarrow{p_A'} - \vec{p}_A = -\overrightarrow{p_B'} + \vec{p}_B$$

$$\overrightarrow{p_A'} + \overrightarrow{p_B'} = \vec{p}_A + \vec{p}_B$$

$$\boldsymbol{Final\ momentum = initial\ momentum}$$

Equilibrium of a particle

What is resultant force?
Resultant force is that single force which produces the same effect on the body as the net effect of all the forces together.

What is equilibrium of forces?
A set of forces are said to be in equilibrium if their resultant is zero.

What is equilibrant force?
The equilibrant is that force which when acts together with those forces keep the body in equilibrium.

What is equilibrium of a particle?
The particle is said to be in equilibrium if the net external force acting on the particle is zero.

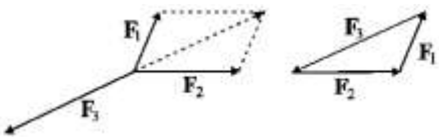
When do we say that the particle is in equilibrium under the action of two forces? Explain.
Two forces on the particle must be equal and opposite.

Let two forces, $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ act on a particle.
The particle will be in equilibrium, if $\overrightarrow{F_1} + \overrightarrow{F_2} = 0$ **OR** $\overrightarrow{F_1} = -\overrightarrow{F_2}$

When do we say that the particle is in equilibrium under the action of three forces? Explain.

Let three forces, $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ act on a particle.

The particle will be in equilibrium, if $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$



When do we say that the particle is in equilibrium under the action of several forces? Explain.

A particle is in equilibrium under the action of several forces, if the resultants of the resolved components of these forces in each of the X and Y-directions are independently zero.

$\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$

Keep in mind

* Isolated system is a system with no external force acts on it.
* For a particle to be in equilibrium, minimum number of forces acting on a particle must be two.

Common forces in mechanics

Mention the Common forces in mechanics?

* Gravitational force
* Spring force
* Tension in the string

What is gravitational force?

It is the force of attraction between the two bodies due to their masses.

What is weight?

The force exerted by the earth on the object is called the weight of the object.

Mention the expression for weight of body.

It is given by, $W = mg$.

Keep in mind

* Every object on the earth experiences the force of gravity due to earth and it can act at a distance without need of material medium.
* Weight is vector quantity and its unit is *newton*.

What are the differences between Mass and Weight OR Distinguish between mass and weight.

Mass	Weight
It is the amount of matter contained in a body	It is the gravitational force of attraction on a body
It is a scalar	It is vector
Mass of the body remains same at all places	Weight of the body varies from place to place
SI unit is kilogram	SI unit is newton

What is spring force?

When a spring is compressed or extended by an external force a restoring force is generated. This restoring force is called spring force.

Mention the expression for spring force and explain the terms.

The spring force is given by, $F = -kx$

where $k \rightarrow$ spring constant and $x \rightarrow$ Displacement.

The $-ve$ sign denotes that the force is opposite to the displacement.

What is tension in a string?

The restoring force in a string is called tension.

For an inextensible string the force constant is very high.

What are contact forces? Give examples.

When two bodies are in contact then they exert force on each other. These forces are called as contact forces.

Ex: Spring force, tension in a string, buoyant force, viscous force and air resistance etc.

In mechanics we come across so many contact forces, their origin is electrical force though the particles are neutral. Explain.

When bodies are in contact, there are mutual contact forces. They are in accordance with Newton's third law. All the contact forces are electrical in nature. At microscopic level all the matter consists of charged particles namely, electrons and protons. The contact forces between objects in contact arising due to elasticity of bodies, molecular collisions and impacts etc.

Keep in mind

* When bodies are in contact, there are mutual contact forces.
* They are in accordance with Newton's third law.
* The contact forces between objects in contact arising due to elasticity of bodies, molecular collisions and impacts etc.

What s normal reaction (N)?

The component of contact force normal to the surface in contact is called Normal reaction.

What is friction (f)?

The component of contact force parallel to the surface in contact is called Friction.

Friction opposes impending or relative motion between the two surfaces.

Mention the types of friction.

* Static friction
* Kinetic friction

What is static friction (f_s)?

Static friction is the force which balances the applied force when a body is in the state of rest.

Why static friction is called self-adjusting force? Explain.

When there is no applied force, there is no static friction. It comes to play at that moment when there is an applied force. As the applied force increases, static friction also increases and remains equal and opposite to applied force up to a certain limit. Hence it is called a self-adjusting force.

What is limiting friction?

The maximum static friction that a body can exert on the other body in contact with it is called limiting friction.

.

Mention the expression for limiting friction.

The limiting friction is directly proportional to the normal reaction between the two surfaces.

That is, $(f_s)_{max} \propto N$.

$$\boldsymbol{(f_s)_{max} = \mu_s N}$$

where $\mu_s \rightarrow$ co-efficient of static friction and it has no unit.

What is kinetic friction? Mention the expression for it.

Frictional force that opposes the relative motion between the surfaces in contact is called kinetic friction.

$$\boldsymbol{f_k = \mu_k N}$$

where $\mu_k \rightarrow$ co-efficient of kinetic friction and it has no unit.

Keep in mind

When the body begins, the force acting on the body is given by, $F - f_k = ma$.

Case(1): If velocity is constant then, $a = 0$, $F - f_k = 0$ then $F = f_k$

Case(2): If the applied force is removed, $F = 0$ then, $-f_k = ma$

$a = -\frac{f_k}{m}$, the body eventually comes to rest.

State laws of friction.

(1) The direction of static friction is opposite to the impending motion and the magnitude is given by, $f_s \leq \mu_s N$
(2) The direction of kinetic friction is opposite to relative motion of the body and the magnitude is given by, $f_k = \mu_k N$
(3) The values of μ_s and μ_k depend on the nature of the surfaces and μ_k is generally less than μ_s
(4) The coefficients of friction are independent of area of contact, provided normal force is constant.
(5) Kinetic friction is nearly independent of velocity.

What is rolling friction? Explain.

The force which opposes the rolling motion of a body is called rolling friction.

In principle, a body like ring rolling without slipping over a horizontal plane will suffer no friction. But in practice some resistance to motion does occur. Rolling friction has a complex origin and somewhat different from that of static and kinetic friction. Rolling friction is much smaller than these.

Mention the advantages of friction.

(1) Friction helps in walking on ground.
(2) Brakes of vehicle work on account of friction.
(3) Writing with chalk on the black board is possible because of friction.
(4) Nails and screws can be fixed an account of friction.
(5) A match stick is lighted due to friction.
(6) Moving belt remains on the rim of wheel because of friction.

Mention the disadvantages of friction.

(1) Friction causes wear and tear of machine parts.
(2) Efficiency of the machine is reduced on account of friction.
(3) Heat is generated because of friction that may damage the machinery.
(4) Friction restricts the speed of the vehicles.

Mention the methods of reducing friction.

(1) By polishing the surfaces.
(2) Using lubricants like oil, grease etc. in machines.
(3) Using the materials of low co-efficient of friction.
(4) Using ball bearing in wheels, axels and shafts of automobiles.
(5) By providing the streamlined shape to moving vehicles (car, bus, aeroplanes etc.).

Circular motion

What is circular motion?

Motion of a body in circular path is known as circular motion.

Mention the expression for the acceleration in circular motion.

In circular motion, the moving body possess an acceleration which is directed towards the centre of the circular path is given by,

$$a = \frac{v^2}{R} \qquad \text{where } R \text{ is the radius of the circular path}$$

What is centripetal force?

The force which is directed towards the centre of the circular path is called centripetal force.

Derive the expression for centripetal force?

Force acting on the body is given by, $f_c = ma$

where $m \rightarrow$ mass of the body executing circular motion and $a \rightarrow$ centripetal acceleration

$$\text{But} \quad a = \frac{v^2}{R}$$

$$\text{Then,} \quad f_c = m\frac{v^2}{R}$$

$$\boldsymbol{f_c = \frac{mv^2}{R}}$$

Give examples for centripetal force.

(1) When a stone is rotated in a circle by a string, the centripetal force is provided by the tension in the string.
(2) The centripetal force for the motion of planet around the sun is provided by gravitational force on the planet due to the sun.
(3) When a car takes turn on a horizontal road, the centripetal force is the force of friction.

Mention the forces acting on moving car on level road.

Forces acting on the car are,

* The weight of the car, mg
* Normal reaction, N
* Friction, f

Derive the expression for Maximum speed on the level road.

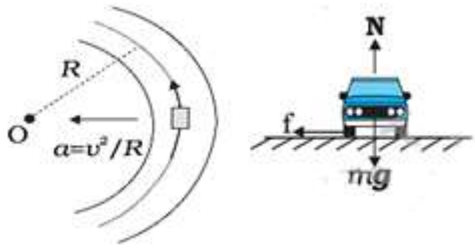
There is no acceleration in the vertical direction;

$$N - mg = 0$$
$$N = mg$$

Since the car is moving in circular path, the centripetal force is provided by friction.

$$f_s = f_c$$

For maximum speed, $(f_s)_{max} = \frac{mv_{max}^2}{R}$



$$\mu_s N = \frac{mv_{max}^2}{R}$$
$$v_{max}^2 = \frac{\mu_s RN}{m}$$
$$v_{max}^2 = \frac{\mu_s R(mg)}{m} = \mu_s Rg$$
$$\boldsymbol{v_{max} = \sqrt{\mu_s Rg}}$$

What is banking of road?

Rising of outer edge of the road as compared to the inner edge to provide centripetal for vehicles is called banking of roads.

What is need of banking of roads?

We can reduce the contribution of friction in circular motion of the car if the road is banked.

What is angle of Banking (θ)?

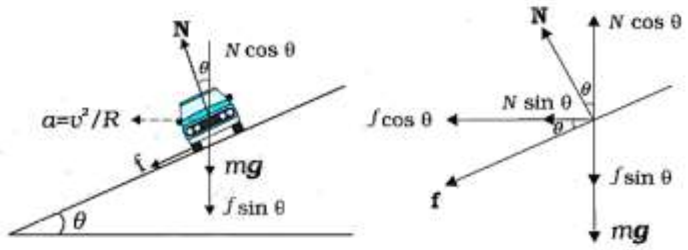
The angle through which the outer edge of the road is raised as compared to the inner edge is called angle of banking.

Derive the expression for Maximum speed on the banked road.

As there is no acceleration in the vertical direction,

$$N\cos\theta = f\sin\theta + mg$$
$$N\cos\theta - f\sin\theta = mg \qquad \rightarrow (1)$$

The centripetal force is provided by horizontal components of N and f



$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \qquad \rightarrow (2)$$

Taking equation (2) ÷ (1),

$$\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = \frac{mv^2}{R} \times \frac{1}{mg}$$
$$\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = \frac{v^2}{Rg}$$

Taking N as common in LHS,

$$\frac{N\left(\sin\theta + \frac{f}{N}\cos\theta\right)}{N\left(\cos\theta - \frac{f}{N}\sin\theta\right)} = \frac{v^2}{Rg}$$
$$\frac{\left(\sin\theta + \frac{f}{N}\cos\theta\right)}{\left(\cos\theta - \frac{f}{N}\sin\theta\right)} = \frac{v^2}{Rg}$$

To obtain maximum speed we put $\frac{f}{N} = \mu_s$

$$\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta} = \frac{v_{max}^2}{Rg}$$

$$\frac{v_{max}^2}{Rg} = \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$$

Divide RHS of numerator and denominator by $\cos\theta$

$$\frac{v_{max}^2}{Rg} = \left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)$$

$$v_{max}^2 = Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)$$

$$\boldsymbol{v_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)}}$$

This is the maximum speed, that a car can take turn without slipping in a circular path.

Mention the expression for optimum speed of a car on banked road.

The optimum speed to negotiate a curve can be obtained by putting $\mu_s = 0$

$v_0 = \sqrt{Rg\tan\theta}$ and banking angle, $\theta = \tan^{-1}\left(\frac{v_0^2}{Rg}\right)$

Mention the steps in solving problems in mechanics.

(1) Draw the free body diagram showing a given body as a point.
(2) Consider a body of interest in a given problem and mark as a point. If the given problem has two bodies, then mark then as different points.
(3) Mark the various forces acting on each body.
(4) Write the equation of motion and solve the given unknown parameters.

Chapter 5 **WORK, POWER AND ENERGY**

Scalar product or dot product of two vectors

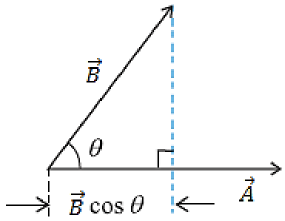
Define scalar product of two vectors OR Define dot product of two vectors.

The Scalar product of two vectors is defined as the product of the magnitude of the first vector and the component of second vector in the direction of first vector.

Write the mathematical equation for scalar (dot) product of two vectors.

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$ denoted by $\vec{A}\cdot\vec{B}$ is given by

$$\boldsymbol{\vec{A}\cdot\vec{B} = AB\cos\theta}$$



Where $\theta \rightarrow$ angle between the two vectors

$\vec{A}\cdot\vec{B} = (magnitude\ of\ \vec{A}) \times (magnitude\ of\ \vec{B}\ along\ \vec{A})$

Explain how commutative law holds well in dot product.

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{B}\cdot\vec{A} = BA\cos\theta = AB\cos\theta$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Dot product is commutative.

Explain how distributive law holds well in dot product.

$\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = A(B+C)\cos\theta = AB\cos\theta + AC\cos\theta$

$\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

$\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

Dot product is distributive over the addition of vectors.

What is the value of scalar product of two equal vectors?

Let $\vec{A}$ and $\vec{B}$ be two parallel vectors. The angle between two parallel vectors is zero.

$\vec{A}\cdot\vec{B} = \mathrm{AB}\cos 0$

$\vec{A}\cdot\vec{B} = AB \qquad (\cos 0 = 1)$

If $\vec{A}\cdot\vec{B} = AB$ then, what is the angle between $\vec{A}$ and $\vec{B}$?

If $\vec{A}\cdot\vec{B} = AB$ then, angle between them is zero and given vectors are parallel.

What is the value of scalar product of a vector with itself?

Let $\vec{A}$ and $\vec{B}$ be two equal vectors. The angle between two equal vectors is zero.

$\vec{A}\cdot\vec{A} = AA\cos 0 = A^2$

What is the value of dot product of unit vector with itself?

$\hat{\imath}\cdot\hat{\imath} = 1\times 1\times\cos 0 = 1$

$\hat{\jmath}\cdot\hat{\jmath} = 1\times 1\times\cos 0 = 1$

$\hat{k}\cdot\hat{k} = 1\times 1\times\cos 0 = 1$

What is the value of scalar product of two mutually perpendicular vectors?

If two vector $\vec{A}$ and $\vec{B}$ are perpendicular, then angle between them is $\theta = 90^0$

$\vec{A} \cdot \vec{B} = AB \cos 90^0$

$\vec{A} \cdot \vec{B} = 0$

What is the condition for the two vectors to be perpendicular to each other?

The two vectors are perpendicular to each other, if their dot product is zero.

If $\vec{A} \cdot \vec{B} = 0$ then, what is the angle between $\vec{A}$ and $\vec{B}$?

If $\vec{A} \cdot \vec{B} = 0$ then, angle between them is 90^0 and given vectors are perpendicular to each other.

What is the value of dot product of two mutually perpendicular unit vectors?

$\hat{\imath} \cdot \hat{\jmath} = 1 \times 1 \times \cos 90^0 = 0$

$\hat{\jmath} \cdot \hat{k} = 1 \times 1 \times \cos 90^0 = 0$

$\hat{k} \cdot \hat{\imath} = 1 \times 1 \times \cos 90^0 = 0$

What is the value of scalar product of two anti-parallel vectors?

If $\vec{A}$ and $\vec{B}$ are two anti parallel vectors, angle between them is 180^0.

$\vec{A} \cdot \vec{B} = AB \cos 180^0$

$\vec{A} \cdot \vec{B} = -AB \qquad (\cos 180^0 = -1)$

If $\vec{A} \cdot \vec{B} = -AB$ then, what is the angle between $\vec{A}$ and $\vec{B}$?

If $\vec{A} \cdot \vec{B} = -AB$ then, angle between them is 180^0 and given vectors are antiparallel vectors.

Obtain the expression for dot product of two vectors in terms of their components.

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$\vec{A} \cdot \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

$\vec{A} \cdot \vec{B} = A_xB_x(\hat{\imath} \cdot \hat{\imath}) + A_xB_y(\hat{\imath} \cdot \hat{\jmath}) + A_xB_z(\hat{\imath} \cdot \hat{k}) + A_yB_x(\hat{\jmath} \cdot \hat{\imath}) + A_yB_y(\hat{\jmath} \cdot \hat{\jmath}) + A_yB_z(\hat{\jmath} \cdot \hat{k})$
$\quad + A_zB_x(\hat{k} \cdot \hat{\imath}) + A_zB_y(\hat{k} \cdot \hat{\jmath}) + A_zB_z(\hat{k} \cdot \hat{k})$

But $\quad \hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$ and

$\quad \hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$

$\therefore \boldsymbol{\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z}$

Obtain the expression for angle between vectors.

Let θ be the angle between A and B, then $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$\boldsymbol{\theta = cos^{-1}\left[\frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}\right]}$$

Keep in mind

* There are two ways of multiplying vectors. One way known as the ***scalar product*** gives a ***scalar*** from two vectors. The other known as the ***vector product*** gives a new ***vector*** from the two vectors.

Illustration

If the magnitude of two vectors are 4 and 6 and the magnitude of their scalar product is $12\sqrt{2}$, what is the angle between the vectors?

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{12\sqrt{2}}{4\times 6} = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^0$$

Work and work energy theorem

What do you mean by work done by a force?

Work is said to be done when a force applied on a body displaces the body through a certain distance in the direction of applied force.

Define work done by a constant force.

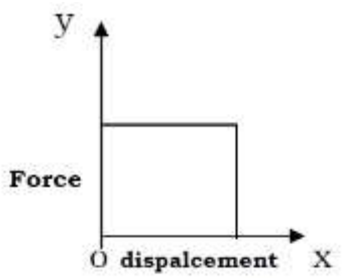
The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.

$\boldsymbol{W = Fd\cos\theta = \vec{F}\cdot\vec{d}}$

Mention the SI unit and dimensions of work.

The SI unit of work is *joule (J)* and dimensions are $[ML^2T^{-2}]$

How do you represent graphically work done by a constant force?



Under what conditions the work done by a force is maxi mum and minimum?

Work done by a force is maximum, when the force and displacements are in the same direction and minimum when they are perpendicular to each other.

Explain the nature of work done with examples.

(i) Positive work: The force and the displacement are in the same direction. i.e. θ is less than 90^0.

Ex: (1) When a spring is stretched, force acting on the spring and the displacement are in same direction.

(2) When a lawn roller is pulled or pushed by applying a force then the work done is positive.

(ii) Zero work: If displacement is zero, or if the force is zero or if the force and the displacement are mutually perpendicular ($\theta = 90^0$) then the work done is zero.

Ex:(1) A man holding a mass of 50 kg on his head then work done is zero, because d= 0.

(2) A particle moving on a smooth surface which is not acted upon by a horizontal force.

(3) A man holding a suitcase on his head and moves on a horizontal road. Here force on the suitcase is upward and displacement is along Horizontal.

(4) A particle moving in a circle with constant speed the centripetal force is always perpendicular to the displacement.

(iii) Negative work: The force and displacement are in opposite direction.
i.e. θ is greater than 90^0 up to 180^0

Ex: (1) When a body of mass m is raised upwards from the ground through a height h.

(2) When breaks are applied to stop a moving car the work done by the breaking force is negative

State the conditions under which a force does no work.

A force does no work when,

(i) displacement is zero.

(ii) force and the displacement are mutually perpendicular.

Illustration

A body is pushed through 6 m across a surfave offering 60 N resistance. How much work is done by the (a) applied force and (b) resistive force?

(a) $W = Fd\cos\theta = 60 \times 6 \times \cos 0 = 360 \times 1 = 360\ N$

(b) $W = Fd\cos\theta = 60 \times 6 \times \cos 180 = 360 \times (-1) = -360\ N$

What is energy? Mention the forms mechanical energy.

The energy of a body is its capacity or ability for doing work.

Mechanical energy has two types, (i) Kinetic energy and (ii) Potential energy

What s kinetic energy? Give examples.

The energy possessed by a body by virtue of its motion is called kinetic energy.

Ex: Flowing water, moving vehicles, a bullet fired from a gun. Etc.

Obtain the expression for Kinetic energy.

Consider an object of mass m has velocity $\vec{v}$.

The kinetic energy is, $K = \frac{1}{2}\, m\vec{v}\cdot\vec{v}$

$$\boldsymbol{K = \frac{1}{2}\, mv^2}$$

What is SI unit and dimensions of energy?

The SI Unit energy/kinetic energy/potential energy is $joule$ (J). Dimensions are $[ML^2T^{-2}]$

Keep in mind

* Work is a scalar quantity.
* Work done and energy has same dimensions.
* Work done = Area under the force - displacement graph

Mention alternate units of work / Energy.

1. In CGS system 'erg' 1erg = 10^{-7} J
2. Electron volt 1eV = 1.6×10^{-19} J
3. Kilowatt hour 1 kWh = 3.6×10^6 J
4. Calorie 1 Cal = 4.186 J

Illustration

Convert 1 kWh in joule.

$1kWh = 1000\ W \times 3600\ s = 3600000\ W - s = 3.6 \times 10^{-6}\ J$

The energy associated with the daily food intake of a human adult is 10^7 J express it in kilo calories.

$4.186\ J = 1\ cal$

$10^7 J = \dfrac{1}{4.186} \times 10^7 = 0.2389 \times 10^7 = 0.2389 \times 10^3 \times 10^4 = 2389\ k\ cal$

Name the largest and smallest practical unit of energy.

Largest practical unit is kilowatt hour [kWh]

Smallest practical unit is electron volt (eV)

State and prove work energy Theorem for constant force.

The change in kinetic energy of a particle is equal to the work done on it by the net force.

Proof: One of the equations of motion for rectilinear motion is, $v^2 = v_0^2 + 2ax$

By generalizing the equation to three dimensions, we have $v^2 = v_0^2 + 2\vec{a} \cdot \vec{d}$

$$v^2 - v_0^2 = 2\vec{a} \cdot \vec{d}$$

Multiplying both sides by $\frac{1}{2}\ m$, we have, $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}\ m\ 2\vec{a} \cdot \vec{d}$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = m\vec{a} \cdot \vec{d}$$

$$K_f - K_i = \vec{F} \cdot \vec{d}$$

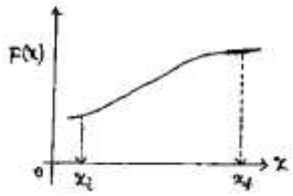
$$\boldsymbol{K_f - K_i = W} \quad \text{or} \quad \boldsymbol{\Delta K = W}$$

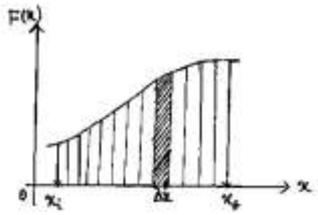
Obtain the expression for work done by a variable force.

Let us consider a variable force (Whose magnitude changes continuously) acting on a body.

Let the body be displaced in the direction of applied force.

The graph of variable force, *F(x)* and displacement, x of the body is as shown.





To calculate the work done, divide the total displacement of the body into a number of small intervals each of width Δx.

The width Δx is so small that the force $F(x)$ is considered constant over that interval.

Then $\Delta W = F(x)\Delta x$

Total work done is given by, $W = \sum_{x_i}^{x_f} \Delta W$

$$W = \sum_{x_i}^{x_f} F(x)\Delta x$$

If displacements are allowed to approach zero, then number of strips increases infinitely, and the sum approaches a definite value.

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\Delta x$$

$$\boldsymbol{W = \int_{x_i}^{x_f} F(x)dx}$$

Prove work energy theorem for variable force.

We know that $K = \frac{1}{2}mv^2$

Differentiating both sides with respect to t,

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{1}{2}\, mv^2\right)$$

$$\frac{dk}{dt} = \frac{1}{2}\, m\, \left(2v\frac{dv}{dt}\right) = m\frac{dv}{dt}\, v$$

$$\frac{dk}{dt} = mav$$

$$\frac{dk}{dt} = Fv = F\frac{dx}{dt}$$

When $x = x_i,\ K = K_i\ \&\ x = x_f, K = K_f$

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F\, dx$$

$$\int_{x_i}^{x_f} F\, dx = \int_{K_i}^{K_f} dK$$

$$W = [K]_{K_i}^{K_f}$$

$$\boldsymbol{W = K_f - K_i}$$

Discuss the nature of work based on work energy theorem with examples.

1. Work done by a Force is zero, if there is no change in the speed of a body.

$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = 0.$ $\qquad (v = v_0)$

Ex: When a body moves in a circular path with constant speed, there is no change in kinetic energy of the body.

2. Work done by a Force is positive, if there is increase in the velocity of the particle.

$W = \frac{1}{2}m\,(v - v_0)$

If $v > v_0, v - v_0 = +ve,$ W = positive

Ex: When a particle is dropped from the top of the building the velocity of the particle increases.

3. Work done by a force is negative, if there is decrease in the speed of the particle

$W = \frac{1}{2}m\,(v - v_0)$

If $v > v_0,\ v - v_0 = -ve,$ W = negative

Ex: When particle is projected upwards, the speed of the particle decreases, work done is negative.

Mention the relation between kinetic energy and linear momentum

We have $K = \frac{1}{2}\, mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{P^2}{m}$ $\qquad (mv = P)$

Concept of Potential Energy

What is potential energy? Give examples.

The energy possessed by a body by virtue of its position or configuration (shape) is called potential energy.

Ex:

1. An object lifted to a certain height from the surface of the earth has potential energy at the position.
2. A stretched bow and arrow system has potential energy.
3. A wound spring of a watch has potential energy.
4. An apple/mango hanging from the branch of a tree has a potential energy.

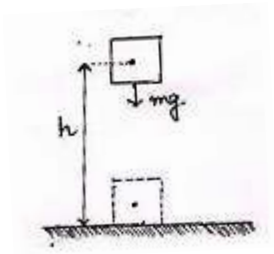
Keep in mind

* In the definition of potential energy position refers to the height above the surface of earth and configuration refers to arrangement/shape of the body.
* Potential energy is a stored energy when work is done on that body.

Derive the expression for Potential energy?

Consider a block of mass m which is to be raised to a height h above the ground.

Work done by the External force is,



$$W = \vec{F_e} \cdot \vec{h} = F_e h \cos\theta$$

$$W = mgh \cos 0$$

$$W = mgh$$

This work done is stored as potential energy $V(h)$.

$$\boldsymbol{V(h) = mgh}$$

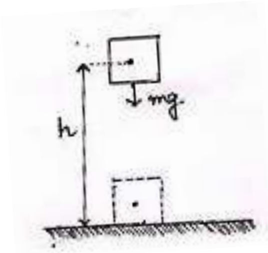
What is gravitational Potential energy?

Work done by the gravitational force on the body when it is raised to a certain height is known as gravitational potential energy and denoted by $V(h)$ as function of the height h.

Derive the expression for Gravitational potential energy.

Consider a block of mass m which is to be raised to a height h above the ground.

Work done by the Gravitational force is,



$$W = \vec{F_g} \cdot \vec{h} = F_g h \cos\theta$$

$$W = mgh \cos 180^0$$

$$W = -mgh$$

This work done is stored as gravitational potential energy $V(h)$.

$$\boldsymbol{V(h) = -mgh}$$

Here negative sign indicates that gravitational force acts downwards.

Mention the types of Potential energy.

(i) Gravitational potential energy: The energy passed by a body by virtue of its position.

(ii) Elastic potential energy: The energy possessed by a body by virtue of its deformed shape.

If an object of mass 'm' is released from rest form the top of a frictionless inclined plane of height 'h', what is its speed at the bottom of the inclined plane.

The object comes down with an increasing speed and its speed is given by the equation,

$v^2 = v_0^2 + 2gh$

$v^2 = 2gh$

$\boldsymbol{v = \sqrt{2gh}}$

Show that the potential energy of the object at height *h* when the object is released manifests itself as kinetic energy of the object on reaching the ground.

The object comes down with an increasing speed and its speed is given by the equation,

$$v^2 = v_0^2 + 2gh$$

$$v^2 = 2gh$$

Multiplying both sides with $\frac{m}{2}$, $\frac{1}{2}mv^2 = \frac{m}{2} \times 2gh$

$$\frac{1}{2}mv^2 = mgh$$

This shows that the potential energy of the object at height h when the object is released manifests itself as kinetic energy of the object on reaching the ground.

Show that the work done by a conservative force is equal to negative of change in gravitational potential energy.

$$V(x) = -mgx$$

$$\frac{dV(x)}{dx} = -mg$$

$$-\frac{dV}{dx} = F(x)$$

This implies that, $F(x)\,dx = -dV$

On integration $\int_{x_i}^{x_f} F(x)\,dx = -\int_{V_i}^{V_f} dV = V_i - V_f$

Law of conservation of mechanical energy

What is conservative force? Give examples.

If the amount of work done by or against a force depends only on the initial and final positions of a body and not on the path followed by the body then such a force is called a conservative force.

Ex: Gravitational force, spring force and electrostatic force are conservative forces.

What is non conservative force? Give examples.

It the amount of work done against a force depends on the path followed by a body then the force is said to be non-conservative force.

Ex: Frictional force and viscous force are non-conservative forces.

Distinguish between Conservative force and Non conservative force.

Conservative force	Non-conservative force
Work done depends on initial point and final point	Work done depends on path followed by the body
Work done is zero around a closed path	Work done is not-zero around a closed path
Work done is path independent	Work done is path dependent

State and Explain law of conservation of mechanical energy.

The total mechanical energy of a system is conserved, if the forces doing the work on it are conservative.

Explanation:

Suppose a body undergoes displacement dx under the action of a conservative force, F.

Then from work energy theorem $K_f - K_i = \int_{x_i}^{x_f} F(x)dx$

The potential energy $V(h)$ is defined by the force F can be written as, $\int_{x_i}^{x_f} F(x)dx = V_i - V_f$

From the above equations we get, $K_f - K_i = V_i - V_f$

$$K_i + V_i = K_f + V_f$$

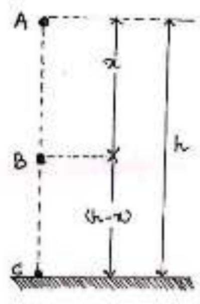
Thus Initial mechanical energy of a system is equal to final mechanical energy of system.

Give an illustration for the conservation of mechanical energy in case of a ball dropped from a cliff of height 'h'? <u>OR</u>

Illustrate law of conservation of mechanical energy in the case of freely falling body.

In case of freely falling body mechanical energy $(K + V)$ of the body remains constant.

At point A:



Consider a body of mass m having $v_0 = 0$ at a height h from the ground.

The kinetic energy is, $K = \frac{1}{2}mv_0^2 = 0$ $(\because v_0 = 0)$

The potential energy is $V = mgh$

$\therefore$ Mechanical energy at A, $K + V = 0 + mgh = mgh --- (1)$

At point B:

Let the body is allowed to fall. It reaches to B travelling a distance x with a velocity v_B.

Then $AB = x$ and $BC = (h - x)$

The potential energy is, $V = mg\,(h - x)$

The velocity attained by the body is, $v_B^2 = v_0^2 + 2gx$

$$v_B^2 = 2gx$$

$\therefore$ The kinetic energy is $K = \frac{1}{2}\,mv_B^2$

$$K = \frac{1}{2}m \times 2gx = mgx$$

Mechanical energy at B is $(K+V) = mgx + mg(h - x)$

$$= mgx + mgh - mgx$$

$$= mgh --- (2)$$

At point C:

Now the body reaches to the ground at C.

Here $h = 0$, then potential energy, $V = mg \times 0 = 0$.

The velocity attained by the body just reaches the point C, $v_c^2 = v_0^2 + 2gh$

$$v_C^2 = 2gh$$

The kinetic energy is $K = \frac{1}{2}\,mv_c^2$

$$K = \frac{1}{2}m \times 2gh$$

$$K = mgh$$

Mechanical energy at C is $(K + V) = mgh + 0 = mgh --- (3)$

From the equations (1), (2) and (3), it is clear that, the mechanical energy of a body during the free fall of a body under the action of gravity remains constant.

Potential energy of spring

State force law for spring.

When a spring is compressed or stretched, the spring force is given by,

$F_s = -kx$ where k is spring constant.

This force law for spring is called Hooke's law.

Define spring constant.

Force constant or spring constant is the restoring force per unit displacement of the spring.

$$\boldsymbol{k = \frac{F}{x}}$$

What it represent, if the spring constant of a given spring is (a) large and (b) small?

(a) The spring is said to be stiff (b) The spring is said to be soft (or smooth)

Mention the SI unit and dimensions of spring constant.

Its SI unit is Nm^{-1}. Dimensions are ML^0T^{-2}

Show that spring force is conservative.

Work done by spring force, $W = \int_0^{x_m} F_s\,dx = \int_0^{x_m} -kx\,dx$

If the spring is stretched from initial displacement x_i to final displacement x_f then,

$$W = -\int_{x_i}^{x_f} kx\,dx = -k\int_{x_i}^{x_f} x\,dx$$

$$W = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = -\frac{1}{2}k\left[x_f^2 - x_i^2\right]$$

If the spring is pulled from x_i and allowed to return to x_i then,

$$W = -\frac{1}{2}k\left[x_i^2 - x_i^2\right]$$

$$W = -\frac{1}{2}k[0] = 0$$ Thus spring force is Conservative.

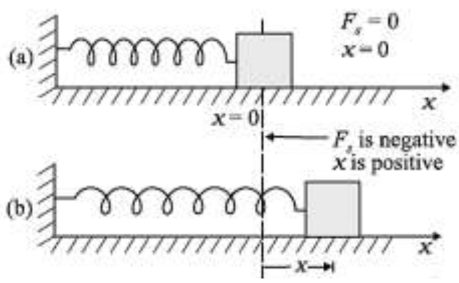
What is potential energy of a spring?

Work done by the spring force when it is compressed or stretched is stored as energy called potential energy.

Derive the expression for potential energy of a spring.

Let a block attached to a light (mass less) spring and resting on a smooth horizontal surface. Let the spring is stretched through a displacement x_m.

Work done by spring force is, $W = \int_0^{x_m} F_s\,dx = \int_0^{x_m} -kx\,dx$



$$W = -k\int_0^{x_m} x\,dx$$

$$W = -k\left[\frac{x^2}{2}\right]_0^{x_m} = -\frac{1}{2}k\left[x_m^2 - 0\right]$$

$$W = -\frac{1}{2}\, kx_m^2$$

The work done is stored as the potential energy of the stretched string. $\boldsymbol{V(x) = -\frac{1}{2}kx^2}$

Keep in mind

(1) The same is true when the spring is compressed $\boldsymbol{V(x)} = -\frac{1}{2}kx_c^2$

(2) The work done by the external force is positive and $V(x) = \frac{1}{2}kx^2$

Derive the expression for Kinetic Energy of spring.

The total mechanical energy of a spring at any arbitrary point x, where x lies between $-x_m$ and $+x_m$ will be given by,

$$\text{Total mechanical energy} = V + K$$

$$\frac{1}{2}\, kx_m^2 = \frac{1}{2}\, kx^2 + K$$

$$K = \frac{1}{2}kx_m^2 - \frac{1}{2}kx^2$$

$$\boldsymbol{K = \frac{1}{2}k\left(x_m^2 - x^2\right)}$$

Derive the expression for maximum speed of spring.

When $x = 0$, $K_{max} = \frac{1}{2}\, kx_m^2$

Further, $\frac{1}{2}mv_{max}^2 = \frac{1}{2}\, kx_m^2$

$$\boldsymbol{v_{max} = \left(\sqrt{\frac{k}{m}}\right) x_m}$$

$\boldsymbol{\frac{k}{m}}$ has the dimension of $[T^{-2}]$

Discuss the variation of Potential energy and kinetic energy during elongation and compression of spring.

(1) We have $K = \frac{1}{2}k\,(x_m^2 - x^2)$

When $x = 0$, $K = \frac{1}{2}k\,x_m^2$ $\Rightarrow$ Kinetic energy is maximum at mean position

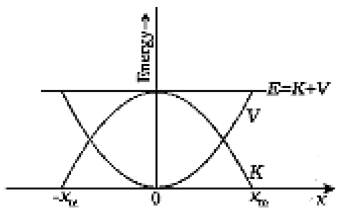
But, $V = \frac{1}{2}kx^2 = \frac{1}{2}k(0) = 0$ $\Rightarrow$ Potential enegy is minimum at mean position

(ii)When $x = x_m$, $K = \frac{1}{2}k\,(x_m^2 - x_m^2) = 0$ $\Rightarrow$ Kinetic energy is minimum at extream position

But, $V = \frac{1}{2}kx_m^2$ $\Rightarrow$ Potential enegy is maximum at extream position

Explain the variation of mechanical energy of spring and represent graphically.

The total mechanical energy can be graphically represented as shown. Kinetic energy is maximum at normal position and potential energy is zero or total energy at normal position is purely kinetic and at extreme ends the total energy is purely potential energy.



Power

What is power?

It is defined as the time rate at which work is done or time rate at which energy is transferred.

Define average power?

$$Average\ power = \frac{work\ done}{time\ taken}$$

But work done = energy supplied or consumed.

$$\therefore\ P_{av} = \frac{Energy}{time\ taken}$$

$$\therefore\ \boldsymbol{P_{av} = \frac{W}{t} = \frac{E}{t}}$$

Define instantaneous power.

It is the limiting value of the average power as the time interval approaches zero.

$$P = \lim_{t \to 0} P_{av}$$

$$\boldsymbol{P = \frac{dW}{dt}}$$

Show that the power is equal to the dot product of force and velocity.

$We\ have,\quad P = \lim_{t \to 0} P_{av}$

$$\boldsymbol{P = \frac{dW}{dt}}$$

But $dW = \vec{F} \cdot d\vec{s}$ where $d\vec{s} \to$ diplacment

$$\therefore P = \vec{F} . \frac{d\vec{s}}{dt}$$

$$\boldsymbol{P = \vec{F} \cdot \vec{v}}$$

If the Force $\vec{F}$ acts in the direction of motion then $\theta = 0, \cos 0 = 1$

$$\therefore P = Fv \cos\theta = Fv \cos 0$$

$$\boldsymbol{P = Fv}$$

What is SI unit and dimensions of Power?

SI unit of power is *watt (W)*, its dimensions are [ML^2T^{-3}]

Define one watt.

The power is said to be *1 watt* if *one joule* work is done/energy is consumed in one second by any agent.

Mention the practical unit of power and write relation between practical unit and SI unit.

The practical unit of power is *horse power (hp)*. *1 horse power (hp) = 746 W*

Collisions

What are collisions?

The term collision refers to the interaction between two bodies due to which the direction and magnitude of the velocity of the colliding bodies change.

Explain the types of collisions with examples.

Elastic collisions: In a collision, if both the linear momentum and kinetic energy of the system are conserved then such a collision is called elastic collision.

It means, the linear momentum and the kinetic energy of the system before and after the collision are same.

Ex: (i) Perfectly elastic collision are rare event, collisions between atomic particles are nearly elastic.

(ii) A ball dropped from a certain height will rebound to the same height, if the collision with the surface is elastic.

Inelastic (plastic) collision: A collision is said to be inelastic if the linear momentum of the system remains conserved but its kinetic energy is not conserved.

In inelastic collision, the loss of kinetic energy appears in the form of heat, elastic potential energy sound and light energy.

Ex: collisions between macroscopic bodies are inelastic A ball dropped from a certain height will not rebound to the some height, if the collision with the surface is inelastic.

What is meant by perfectly inelastic collision? Give example.

A collision is said to be perfectly inelastic if the two bodies after collision stick together and move as one body.

Ex: When a moving bullet hits the stationary wooden block, then it is embedded into the wooden block and both move as one body.

Write the difference between elastic and inelastic collisions

Elastic collision	Inelastic collision
• Both the momentum and kinetic energy are conserved	• Momentum is conserved but the kinetic energy
• Force involved in the collision are conservative	• Forces involved are non-conservative
• Mechanical energy is not converted into other forms of energy	• Mechanical energy is converted into other forms of energy

What is meant by collisions in one Dimension or head-on collision?

If the initial velocities and final velocities of both the bodies are along the same straight line then it is called a one dimensional collision or Head-on collision.

Derive the expression for loss in Kinetic energy in a completely inelastic collision.

Consider two masses m_1 and m_2.

The particle m_1 is moving with speed u_1 and m_2 is at rest.

After collision both masses m_1 and m_2 stick to each other and move as one body with velocity v.

In this type of collision the linear momentum is conserved.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$m_1u_1 = (m_1 + m_2)v \qquad (v_2 = 0 \Rightarrow m_2u_2 = 0)$$

$$v = \frac{m_1 u_1}{m_1 + m_2}$$

As the final kinetic energy is less than the initial kinetic energy,

The change in kinetic energy is, $\Delta K = \frac{1}{2}\, m u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$

Substituting for v from above equation

$$\Delta K = \frac{1}{2}\, m u_1^2 - \frac{1}{2}(m_1 + m_2).\frac{m_1^2}{(m_1 + m_2)^2}\, u_1^2$$

$$\Delta K = \frac{1}{2}\, m_1 u_1^2 - \frac{1}{2}\,\frac{m_1^2}{(m_1 + m_2)}\, u_1^2$$

$$\Delta K = \frac{1}{2}\, m_1 u_1^2 \left[1 - \frac{m_1}{(m_1 + m_2)}\right]$$

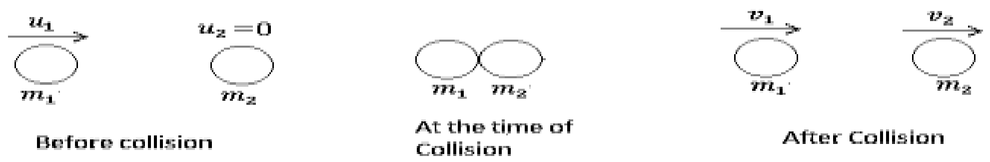
$$\Delta K = \frac{1}{2}\, m_1 u_1^2 \left[\frac{m_1 + m_2 - m_1}{m_1 + m_2}\right]$$

$$\Delta K = \frac{1}{2}\, m_1 u_1^2 \left[\frac{m_2}{m_1 + m_2}\right]$$

$$\boldsymbol{\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\, u_1^2}$$

This loss in kinetic energy appears as the sound and heat energies, Thus total energy is conserved.

Derive expression for final velocities of the bodies in an elastic collision:



Consider two masses m_1 and m_2.

The body m_1 is moving with speed u_1 and m_2 is at rest.

After collision their velocities are v_1 and v_2 respectively and are moving in the same straight line.

As the collision is elastic both the linear momentum and kinetic energies are conserved.

$Momentum \Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$$m_1 u_1 = m_1 v_1 + m_2 v_2 \qquad (m_2 u_2 = 0)$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2$$

$$m_1(u_1 - v_1) = m_2 v_2 \text{ ------ } (1)$$

Kinetic energy $\Rightarrow \frac{1}{2}\, m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_1 v_2^2$

$$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2$$

$$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2 v_2^2 \text{ ------ } (2)$$

Dividing equation (2) by equation (1)

$$\frac{m_1(u_1^2 - v_1^2)}{m_1\,(u_1 - v_1)} = \frac{m_2 v_2^2}{m_2 v_2}$$

$$\frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1\,(u_1 - v_1)} = \frac{m_2 v_2^2}{m_2 v_2}$$

$$u_1 + v_1 = v_2$$

$$v_2 = u_1 + v_1 \;-----\; (3)$$

Substituting equation (3) in equation $m_1 u_1 - m_1 v_1 = m_2 v_2$

$$m_1 u_1 - m_1 v_1 = m_2(u_1 + v_1)$$

$$m_1 u_1 - m_1 v_1 = m_2 u_1 + m_2 v_1$$

$$m_1u_1 - m_2u_1 = m_1v_1 + m_2v_1$$
$$u_1(m_1 - m_2) = v_1(m_1 + m_2)$$
$$\boldsymbol{v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1} \quad ------(4)$$

Substituting (4) in (3)

$$v_2 = u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$
$$v_2 = u_1\left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right]$$
$$v_2 = u_1\left[\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right]$$
$$v_2 = u_1\left[\frac{2m_1}{m_1 + m_2}\right]$$
$$\boldsymbol{v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1} \quad ------(5)$$

The equation (4) and (5) gives the expression for the final velocities of the masses m_1 and m_2 respectively. $i.e\ v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$

Discuss the final velocities of the bodies in an elastic collision in some special cases.

Case (1): If two masses one equal. $i.e\ m_1 = m_2 = m$

$$v_1 = \left(\frac{m - m}{m + m}\right)u_1$$
$$v_1 = 0$$

The final velocity of mass m_1 will be become zero. It means after collision mass m_1 comes to rest and pushes off the second mass m_2.

$$v_2 = \left(\frac{2m}{m + m}\right)u_1 = \frac{2m}{2m}.u_1$$
$$v_2 = u_1$$

Final velocity of mass m_2 = Initial velocity of mass m_1.

It means second mass m_2 move with a velocity of m_1

Case (2): If one the mass dominates. If $m_2 >> m_1$

$$v_2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$
$$as\ m_2 > m_1 \text{ then } m_1 - m_2 = -ve$$
$$v_1 = -u_1$$

Final velocity of mass m_1 get reversed it means it comes back with same velocity.

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$
$$v_2 = \frac{2m_1u_1}{m_2} \approx 0 \qquad \left(becouse\ m_2 \gg m_1, \frac{2m_1u_1}{m_2} \approx 0\right)$$
$$v_2 = 0$$

Heavy mass m_2 practically remains at rest.

Case (3): If $m_1 \gg m_2$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$
$$v_1 = \frac{m_1}{m_1} u_1 \qquad (m_2\ can\ be\ neglected)$$
$$v_1 = u_1$$

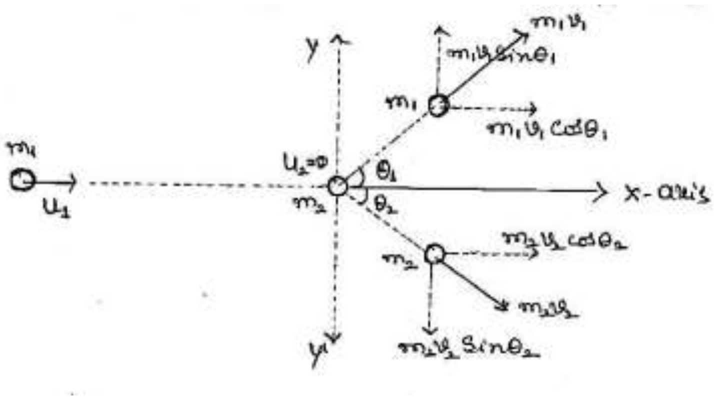
The mass m_1 moves with same velocity.

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1 = \frac{2m_1}{m_1}\ u_1$$

$$v_2 = 2u_1$$

The lighter mass m_2 moves with twice the velocity of the heavy body.

Describe briefly the collisions in two-dimension.



Consider two masses m_1 and m_2.
The body m_1 is moving with speed u_1 and m_2 is at rest.
The mass m_1 collides with the stationary mass m_2 and this is shown in the figure.

After collision, the masses m_1 and m_2 fly-off in different direction. The body m_1 moves with velocity v_1 making an angle θ_1, called deflecting angle, with $x - axis$ and mass m_2 move with a velocity v_2 making an angle θ_2, called angle of recoil, with $x - axis$.

In all types of collision, momentum is conserved.
Therefore conservation of momentum along $x - axis$ and $y - axis$ gives,

(i) $m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\theta_2$ along $x - axis$

(ii) $0 = m_1 v_2 \sin\theta_1 - m_2 v_2 \sin\theta_2$ along $y - axis$

$m_1 v_2 \sin\theta_1 = m_2 v_2 \sin\theta_2$

If the collision is perfectly elastic, then kinetic energy is also conserved.

(iii) $\frac{1}{2}\ m_1 u_1^2 = \frac{1}{2}\ m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Assume that m_1, m_2 and u_1 are known. Now the motion after collision involves four unknowns i.e v_1, v_2, θ_1 and θ_2. To evaluate these four quantities we need fourth equation. However we have only three equations. Therefore fourth equation is to be developed, but this process of developing the fourth equation is quite complicate. To overcome this problem the easiest way of developing fourth equation is the measure the angle of defection θ_1 and the angle of recoil θ_2 experimentally.

Chapter 6 System of particles and Rotational Motion

What is meant by system of particles?
Collection of large number of particles interacting with each other is called system of particles.

What is a rigid Body? Give examples.
Rigid body is a body with a perfectly definite and unchanging shape and the distances between all pairs of particles of such a body do not change even though there are forces on them.

In real situation no body is a perfectly rigid body. However, in bodies like wheels, steel beams, metallic sphere, wooden block etc. deformation under the force is so small that they can be considered as rigid bodies.

What type of motion a rigid body may have when it is not fixed along an axis?
The motion of a rigid body which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation.

What type of motion a rigid body may have when it is fixed along an axis?
The motion of a rigid body which is pivoted or fixed along an axis is rotation.

When does a rigid body said to have translational motion? Give example.
At any instant of time if all the particles of a body have the same velocity then the motion is said to be translational.
Ex: Wooden block sliding on a inclined plane

When does a rigid body said to have rotational motion? Give example.
If every particle of the body moves in a circle which lies in a plane perpendicular to the fixed axis and has its centre on the fixed axis, then the motion is said to be Rotational.
Ex: A ceiling fan

What you mean precession of a spinning body?
The movement of the axis of the rotating body around the vertical axis is termed as precession.

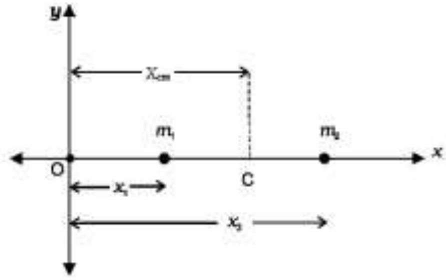
Centre of mass

What is centre of mass of a system of particles?
Centre of mass of a system of particles is the point where the entire mass of the system can be assumed to be concentrated.

Keep in mind

* Any particle lying on the axis of rotation remains at rest while the rigid body rotates about the axis of rotation. Thus axis of rotation is fixed.
* While precession the point of contact of the rotating body with the ground is fixed.
* Centre of mass has the same type of translational motion as the system as a whole if some net external force acts on this *point like mass* as acting on the system.
* Centre of mass of a body or a system is its balancing point.

Mention the expression for centre of mass of a two particle system having masses m_1 and m_2 laying on x-axis.



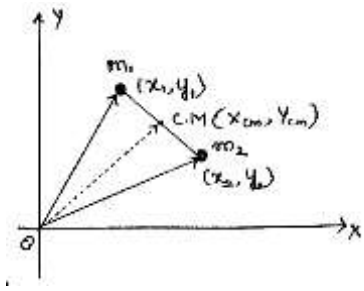
Let the distances of the two particles be x_1 and x_2 respectively from origin.

The masses of the two particles be m_1 and m_2 respectively.

The centre of mass of the system which is at a distance X_{cm} from O is given by, $$\boldsymbol{X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}}$$

If $\vec{R}$ be the position vector of the centre of mass then $\vec{R} = X_{cm}\ \hat{\imath}$

Mention the expression for centre of mass of a two particle system having masses m_1 and m_2 laying in a plane.



If the particles are laying in a plane, we define x and y axes in the plane in which the particle lie and represent the positions of the two particle by co-ordinates (x_1, y_1) and (x_2, y_2) respectively.

The centre of mass located by the co-ordinates (X_{cm}, Y_{cm}) is given by,

$$\boldsymbol{X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}} \quad \text{and} \quad \boldsymbol{Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}}$$

If $\vec{R}$ be the position vector of the centre of mass then $\vec{R} = X_{cm}\ \hat{\imath} + Y_{cm}\hat{\jmath}$

Mention the expression for centre of mass of three particle system laying in a plane.

Let the masses of these particle be m_1, m_2 and m_3 respectively.

Positions of the three particles are represented by co-ordinates (x_1, y_1), (x_2, y_2) and (x_3, y_3).

The centre of mass located by the co-ordinates (X_{cm}, Y_{cm}) is given by,

$$\boldsymbol{X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3y_3}{m_1 + m_2 + m_3}} \quad \text{and} \quad \boldsymbol{Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}}$$

$$\vec{R} = X_{cm}\ \hat{\imath} + Y_{cm}\hat{\jmath}$$

Keep in mind

(1) In a two particle system, laying on x-axis, if the two particles have the same mass $m_1 = m_2 = m$, then

$$X_{cm} = \frac{mx_1 + mx_2}{m + m} = \frac{2n(x_1 + x_2)}{2m} = \frac{x_1 + x_2}{2}$$

Thus for two particles of equal mass the centre of mass lies exactly midway between them.

(2) In a three particle system, laying in a plane, if the particles have equal masses, $m_1 = m_2 = m_3 = m$

$$X_{cm} = \frac{mx_1 + mx_2 + mx_3}{m + m + m} = \frac{m(x_1 + x_2 + x_3)}{3m}$$

$$X_{cm} = \frac{x_1 + x_2 + x_3}{3}$$

$$Y_{cm} = \frac{my_1 + my_2 + my_3}{m + m + m} = \frac{m(y_1 + y_2 + y_3)}{3m}$$

$$Y_{cm} = \frac{y_1 + y_2 + y_3}{3}$$

Thus for three particles of equal mass, the centre of mass coincides with the centroid of the triangle formed by the particles.

Obtain the expression for centre of mass of a system of n-particles.

Consider a system of n-particles.

Let $m_1, m_2 \dots\dots\dots m_n$ be the respective masses of the particles.

Let X_{cm}, Y_{cm} and Z_{cm} are the co-ordinates of the centre of mass of the system.

$$\text{Then} \quad X_{cm} = \frac{m_1x_1 + m_2x_2 + \dots\dots\dots\dots + m_nx_n}{m_1 + m_2 + \dots\dots\dots\dots + m_n}$$

$$X_{cm} = \frac{\sum_{i=1}^{n} m_ix_i}{\sum_{i=1}^{n} m_i}$$

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i$$

Similarly, $Y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i$ and $Z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_iz_i$

The position vector of the centre of mass is, $\boldsymbol{\vec{R} = X_{cm}\hat{\imath} + Y_{cm}\hat{\jmath} + Z_{cm}\hat{k}}$

Further, $\vec{R} = \left(\frac{1}{M}\sum_{i=1}^{n} m_ix_i\right)\hat{\imath} + \left(\frac{1}{M}\sum_{i=1}^{n} m_iy_i\right)\hat{\jmath} + \left(\frac{1}{M}\sum_{i=1}^{n} m_iz_i\right)\hat{k}$

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i\left(x_i\hat{\imath} + y_i\hat{\jmath} + z_i\hat{k}\right)$$

$$\boldsymbol{\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r_i}}$$

Obtain the expression for centre of mass of a rigid body.

For system of n – particles the centre of mass is given by, $\vec{R} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r_i}$

In case of a Rigid body the number of particles is so large that it is impossible to carry out the summation over individual particles in the equation.

Since the spacing of the particle is small, we can treat the body as a continuous distribution of mass.

We subdivide the body into *n* small elements of mass dm_1, dm_2, ...dm_n.

Hence sum $\sum_{i=1}^{n} m_i\vec{r_i}$ can be replaced by the integral $\int dm\ \vec{r}$.

$$\sum_{i=1}^{n} m_i\vec{r_i} = \int dm\ \vec{r}$$

$$\boldsymbol{\vec{R} = \frac{1}{M}\int \vec{r}\, dm}$$

Where $\vec{R} = X_{cm}\,\hat{\imath} + Y_{cm}\hat{\jmath} + Z_{cm}\hat{k}$

Now $X_{cm}\,\hat{\imath} + Y_{cm}\hat{\jmath} + Z_{cm}\hat{k} = \frac{1}{M}\int \left(x\hat{\imath} + x\hat{y} + z\hat{k}\right) dm$

Comparing the co-efficient of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$

$$X_{cm} = \frac{1}{M}\int x\ dm$$

$$Y_{cm} = \frac{1}{M}\int y\ dm$$

$$Z_{cm} = \frac{1}{M}\int z\ dm$$

Keep in mind

If we choose, the centre of mass as the origin of our co-ordinate system, then, $\vec{R} = 0$

implies that, $\int \vec{r} dm = 0$

and $\int x dm = \int y\, dm = \int z\, dm = 0$

Give the location of centre of mass of (i) homogeneous bodies and (ii) bodies having axis of symmetry.

(i) In case of homogeneous bodies like a circular solid disc, an ice cube or a sugar cube, solid sphere, hollow sphere, a marble ball, a billiard ball, an iron ball uniform thin rod etc. The centre of mass coincides with the geometric centers of the bodies.

(ii) In the case of bodies having axis of symmetry like a solid cylinder, hollow cylinder a wheel etc. the centre of mass lies on the axis of symmetry of the body.

Does the centre of mass of a body necessarily lie inside the body? Give examples.

No, it may lie outside the body also.

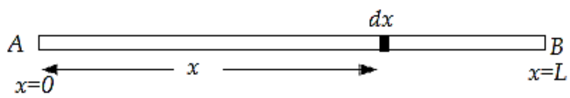
In a solid sphere or solid cube it lies inside the body and in a ring it is outside the body.

Obtain the expression for centre of mass of uniform rod.

Consider a uniform rod of length L.

Let one end A of the rod is taken as the origin.

Since the rod is uniform, the mass per unit length of the rod (λ) is constant.



Consider a small element of the rod of length dx at a distance x from end A and having the mass, $dm = \lambda dx$.

The co – ordinate of the centre of the rod is given by, $X_{cm} = \frac{1}{M}\int_0^L x\, dm$

$$X_{cm} = \frac{1}{M}\int_0^L x\, \lambda dx$$

$$X_{cm} = \frac{1}{M}\ \lambda \int_0^L x dx$$

$$X_{cm} = \frac{1}{M}\ \lambda \left[\frac{x^2}{2}\right]_0^L = \frac{1}{M}\ \lambda \left(\frac{L^2}{2}\right)$$

$$X_{cm} = \frac{1}{M}\ (\lambda L)\left(\frac{L}{2}\right) = \frac{1}{M}\ M\ \left(\frac{L}{2}\right)$$

$$\boldsymbol{X_{cm} = \frac{L}{2}}$$

Illustration

Two bodies of mass 1 kg and 2 kg are laying in XY-plane at (-1,2) and (2,4) respectively. Find centre of mass of two body system.

$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1 \times (-1) + 2 \times 2}{1 + 2} = \frac{-1 + 4}{3} = \frac{3}{3} = 1$

$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{1 \times (2) + 2 \times 4}{1 + 2} = \frac{2 + 8}{3} = \frac{10}{3}$

Co − ordinates are $\left(1, \frac{10}{3}\right)$

Vectorically, $\vec{R} = \hat{\imath} + \frac{10}{3}\hat{\jmath}$

Explain the motion of centre of mass.

Consider a system having n-particles of masses $m_1, m_2, m_3 \dots \dots . m_n$.

Let $\vec{r_1}, \vec{r_2}, \vec{r_3} \dots \vec{r_n}$ be their respective position vectors.

The centre of mass of the system is given by, $\vec{R} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$

$$M\vec{R} = \sum_{i=1}^{n} m_i\vec{r}_i$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots \dots . + m_n\vec{r}_n$$

Differentiating the two sides of the equation with respect to time,

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \dots \dots \dots + m_m\frac{d\vec{r}_n}{dt}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots \dots . . + m_n\vec{v}_n$$

Differentiating the two sides of the equation with respect to time again

$$M\frac{d\vec{V}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + \dots \dots \dots + m_m\frac{d\vec{v}_n}{dt}$$

$$M\vec{A} = m_1\vec{a} + m_2\vec{a}_2 + m_3\vec{a}_3 + \dots \dots . . + m_n\vec{a}_n$$

Using Newton's second law, $\vec{F} = m\vec{a}$

$$M\vec{A} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots \dots \vec{F_n}$$

$\boldsymbol{M\vec{A} = \vec{F}_{ext}}$ where $\vec{F}_{ext} = \vec{F_1} + \vec{F_2} + \dots + \vec{F_n}$

From the above equation we can conclude that, the centre of mass of the system of particles moves as if the mass of the system was concentrated at the centre of mass and all the external force were applied at that point.

Show that the linear momentum of a system of particles is constant when no external force on the system of particles.

Consider a system of n particles of masses $m_1, m_2, m_3, \dots . m_n$ moving with velocities $\vec{v}_1, \vec{v}_2, \vec{v}_3 \dots \dots . . \vec{v}_n$ respectively.

The momentum of the system is given by, $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots + \vec{p}_n$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots \dots . + m_n\vec{v}_n$$

But we have, $M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots \dots . . + m_n\vec{v}_n$

$$M\vec{V} = \vec{P} \text{ or } \vec{P} = M\vec{V}$$

Thus the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of the centre of mass.

Differentiating equation with respect to time

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt}$$

$$\frac{d\vec{P}}{dt} = M\vec{A}$$

But $M\vec{A} = \vec{F}_{ext}$

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$ This is the Newton's second law for system of particles.

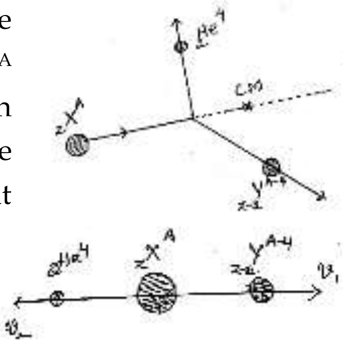
If $\vec{F}_{ext} = 0$ then, $\frac{d\vec{P}}{dt} = 0$ implies that $\vec{P} = constant$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant.

This is the law of conservation of the linear momentum of a system of particles.

Illustrate the motion of center of mass with examples.

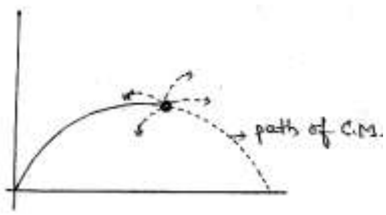
(1) In Radio-active decay the process is caused by the internal force of the system. Therefore initial and final momentums are zero. The Nucleus $_zX^A$ decays into a new nucleus $_{z-2}Y^{A-4}$ and a Helium Nucleus as shown in figure. Since the disintegration occurs only due to the internal force, the nucleus $_{z-2}Y^{A-4}$ and Helium nucleus must move in such a directions that sum of their momentum is zero and their center of mass moves along the path followed by the nucleus $_zX^A$ before decay.



However, when he decay of the nucleus is observed from a frame of reference with respect to which the nucleus is at rest, then the decay products fly off in the opposite directions. The Centre of mass of the system remains at rest. The heavy mass moves with less speed than that of the light mass.

(2) **Explosion of a projectile in midair (fire cracker).**

Let us consider a projectile which explodes in air. Before explosion, the projectile moves along a parabolic path. After explosion each fragment moves along their own parabolic path but the centre of mass of the projectile continues to move in the same parabolic path.



Vector product or cross product of two vectors

Define vector product or cross product of two vectors?

Consider two vectors $\vec{A}$ and $\vec{B}$ such that the angle between them is θ then, the cross product of the vectors $\vec{A}$ and $\vec{B}$ is $\vec{A} \times \vec{B}$ $(\vec{A}\ cross\ \vec{B})$ which is given by,

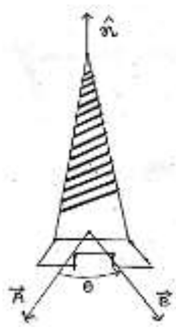
$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = (\boldsymbol{AB}\sin\boldsymbol{\theta})\hat{\boldsymbol{n}}$$ where $\hat{n}$ is the unit vector which gives the direction of $\vec{A} \times \vec{B}$

Give the expression for direction of $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$.

$$\hat{\boldsymbol{n}} = \frac{\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}}{\boldsymbol{AB}\sin\boldsymbol{\theta}} = \frac{\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}}{|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}|}$$

Which is the rule used to determine the direction of $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$.

The direction of $\vec{A} \times \vec{B}$ can be determined by right hand screw rule.



State right hand screw rule used to find the direction of $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$?

Take a right handed screw with its head lying in the plane of $\vec{A}$ and $\vec{B}$ and the screw perpendicular to this plane. If we turn the head of the screw in the direction from A to B through a small angle θ, then the tip of the screw advances in the direction of the vector $\vec{A} \times \vec{B}$.

Mention the difference between Scalar product and vector product.

Scalar product	Vector product
Scalar product of two vector is a scalar	Vector product of two vector is a vector
Scalar product is commutative	Vector product is **not** commutative
Scalar product of two equal or parallel vector is **not equal to zero**	Vector product of two equal or parallel vector is **equal to zero**
Scalar product of two perpendicular vectors is **equal to zero**	Vector product of two perpendicular vectors is **not equal to zero**

Mention the examples for vector product of two vectors.

Moment of a force or torque and angular momentum.

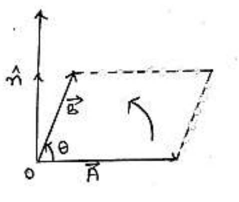
Show that vector product is not commutative.

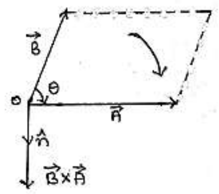
$\vec{A} \times \vec{B} = (AB \sin\theta)\hat{n}$

$\vec{B} \times \vec{A} = (BA \sin\theta)(-\hat{n}) = -(AB \sin\theta)\hat{n}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$





What is the angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$?

Angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ is 180^0 or π radian.

Explain how distributive law holds well in cross product?

Vector product is distributive over vector addition.

$\vec{A} \times (\vec{B} + \vec{C}) = [A(B + C) \sin\theta]\hat{n}$

$\vec{A} \times (\vec{B} + \vec{C}) = [AB \sin\theta + AC \sin\theta]\hat{n}$

$\vec{A} \times (\vec{B} + \vec{C}) = AB \sin\theta\hat{n} + AC \sin\theta\, \hat{n}$

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

What is the value of vector product of two equal vectors?

The angle between the equal vectors is zero

$\vec{A} \times \vec{B} = (AB \sin\theta)\hat{n}$

$\vec{A} \times \vec{B} = (AB \sin 0^0)\hat{n}$ $\qquad (\sin 0^0 = 0)$

$\vec{A} \times \vec{B} = 0$

What is the value of vector product of two parallel vectors?

The angle between the parallel vectors is zero

For equal vectors, $\vec{A} \times \vec{A} = (AA \sin\theta)\hat{n}$

$\vec{A} \times \vec{A} = (AA \sin 0)\hat{n}$

$\vec{A} \times \vec{A} = 0$

What is the value of cross product of unit vector with itself?

$\hat{\imath} \times \hat{\imath} = (1 \times 1 \sin 0)\hat{n} = 0$

$\hat{\jmath} \times \hat{\jmath} = (1 \times 1 \sin 0)\hat{n} = 0$

$\hat{k} \times \hat{k} = (1 \times 1 \sin 0)\hat{n} = 0$

$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

What is the value of cross product of two mutually perpendicular vectors?

The angle between two perpendicular vectors is 90^0

$\vec{A} \times \vec{B} = (AB \sin\theta)\hat{n}$

$\vec{A} \times \vec{B} = (AB \sin 90^0)\hat{n}$ $\qquad (\sin 90^0 = 1)$

$\vec{A} \times \vec{B} = (AB)\hat{n}$

What is the value of cross product of two mutually perpendicular unit vectors?

$\hat{\imath} \times \hat{\jmath} = (1 \times 1 \sin 90^0)\hat{k} = \hat{k}$

$\hat{\jmath} \times \hat{k} = (1 \times 1 \sin 90^0)\hat{\imath} = \hat{\imath}$

$\hat{k} \times \hat{\imath} = (1 \times 1 \sin 90^0)\hat{\jmath} = \hat{\jmath}$

$\hat{\imath} \times \hat{\jmath} = \hat{k}, \quad \hat{\jmath} \times \hat{k} = \hat{\imath}, \quad \hat{k} \times \hat{\imath} = \hat{\jmath}$

Also, $\hat{\jmath} \times \hat{\imath} = (1 \times 1 \sin 90^0)(-\hat{k}) = -\hat{k}$

$\hat{k} \times \hat{\jmath} = (1 \times 1 \sin 90^0)(-\hat{\imath}) = -\hat{\imath}$

$\hat{\imath} \times \hat{k} = (1 \times 1 \sin 90^0)(-\hat{\jmath}) = -\hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \quad \hat{k} \times \hat{\jmath} = -\hat{\imath}, \quad \hat{\imath} \times \hat{k} = -\hat{\jmath}$

Obtain the expression for vector product of two vectors in their rectangular components by analytical method.

The vector $\vec{A}$ and $\vec{B}$ can be written in terms of their rectangular component as,

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

$\vec{A} \times \vec{B} = A_xB_x(\hat{\imath} \times \hat{\imath}) + A_xB_y(\hat{\imath} \times \hat{\jmath}) + A_xB_z(\hat{\imath} \times \hat{k}) + A_yB_x(\hat{\jmath} \times \hat{\imath}) + A_yB_y(\hat{\jmath} \times \hat{\jmath}) + A_yB_z(\hat{\jmath} \times \hat{k})$
$+A_zB_x(\hat{k} \times \hat{\imath}) + A_zB_y(\hat{k} \times \hat{\jmath}) + A_zB_z(\hat{k} \times \hat{k})$

But $\quad \hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

$\hat{\imath} \times \hat{\jmath} = \hat{k},\ \hat{\jmath} \times \hat{k} = \hat{\imath},\ \hat{k} \times \hat{\imath} = \hat{\jmath}$ and $\hat{\jmath} \times \hat{\imath} = -\hat{k},\ \hat{k} \times \hat{\jmath} = -\hat{\imath},\ \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$\vec{A} \times \vec{B} = (A_xB_y)\hat{k} + (A_xB_y)(-\hat{\jmath}) + (A_yB_x)(-\hat{k}) + (A_yB_z)(\hat{\imath}) + (A_zB_x)(\hat{\jmath}) + (A_zB_y)(-\hat{\imath})$

$\boldsymbol{\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}}$

Obtain the expression for vector product of two vectors in their rectangular components by determinant method.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = i \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{\jmath} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_z \\ B_x & B_y \end{vmatrix}$$

$\vec{A} \times \vec{B} = \hat{\imath}(A_yB_z - A_zB_y) - \hat{\jmath}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$

$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$

Magnitude of $\vec{A} \times \vec{B}$ is given by,

$$\boldsymbol{|\vec{A} \times \vec{B}| = \sqrt{(A_yB_z - A_zB_y)^2 + (A_zB_x - A_xB_z)^2 + (A_xB_y - A_yB_x)^2}}$$

Rotational motion

Define angular displacement.

It is defined as angle described by the radius vector in given time

$$Angular\ displacement(\Delta\theta) = \frac{Arc\ length}{Radius} = \frac{x}{r}$$

Mention the SI unit and dimensions of angular displacement.

SI unit of angular displacement is radian (rad). Angular displacement is dimensionless quantity.

Define angular velocity.

It is defined as the ratio of the angular displacement of the particle to the time interval for this displacement.

Define instantaneous angular velocity.

Instantaneous angular velocity is defined as the limit of average angular velocity as time interval approaches to zero

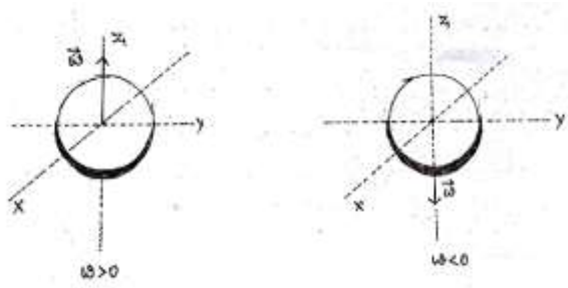
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

How do you recognize the direction of angular velocity?

The direction of angular velocity is along the axis of rotation which can be determined by Right-hand screw rule.

If a body is rotating in the direction of Increasing θ (anticlockwise) then angular velocity of the body is positive.

If the body is rotating in a direction of decreasing θ (clockwise) then angular velocity of the body is negative.



Mention the SI unit and dimensions of angular velocity.

S.I unit of ω is *radian / second (rad - s^{-1})* and Dimensional formula is $M^0L^0T^{-1}$

Derive the vector relation between linear velocity and angular velocity.

Consider a particle at position P of a rigid body.

As the body rotates the particle also moves from position P to position P'

Let its linear displacement $\overline{PP'} = d\vec{r}$ and its angular displacement is $d\theta$.

Now $d\vec{r} = d\theta \times \vec{r}_\perp$

Dividing both sides by *dt,*

$$\frac{d\vec{r}}{dt} = \frac{d\theta}{dt} \times \vec{r}_\perp$$

$\vec{v} = \vec{\omega} \times \vec{r}_\perp$

But from the Δ^{le} OPC, $\vec{r}_\perp = \vec{r} - \overrightarrow{OC}$

Substituting for $\vec{r}_\perp$, $\vec{v} = \vec{\omega} \times (\vec{r} - \overrightarrow{OC})$

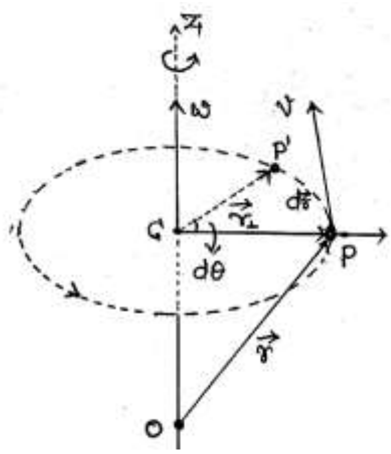
$\vec{v} = \vec{\omega} \times \vec{r} - \vec{\omega} \times \overrightarrow{OC}$

But $\vec{\omega} \times \overrightarrow{OC} = 0$ as $\vec{\omega}$ is along $\overrightarrow{OC}$

$\therefore \vec{v} = \vec{\omega} \times \vec{r} - 0$

$\boldsymbol{\vec{v} = \vec{\omega} \times \vec{r}}$

where $\vec{r} \rightarrow$ position vector of the particle at P.



Keep in mind

* The relation $\vec{v} = \vec{\omega} \times \vec{r}$ shows that the linear velocity of particles situated at different position from axis of rotation is different.

* For the particle laying on the axis of rotation $\vec{r} = 0$. Therefore the linear velocity of the particle at the axis of rotation is zero.
* Angular velocity of every particle of the rigid body is same as that of the angular velocity of the rigid body this is because every particle in the rigid body rotates through the same angle in the same interval of time.

Define angular Acceleration.
The angular acceleration can be defined as the time rate of change of angular velocity.

Define instantaneous angular acceleration.

The instantaneous angular acceleration is given by, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

$$\boldsymbol{\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}}$$

Mention the SI unit and dimensions of angular acceleration.
The unit of angular acceleration is *rad s*$^{-2}$.Dimensional formula is $M^0L^0T^{-2}$

Define moment of force or torque.
Torque acting on a particle is defined as the vector product of the force acting on the particle and the perpendicular distance of the application of force from the axis of rotation of the particle.

Give the expression for torque acting on a particle.
Torque, τ acting on the particle with respect the origin is given by, $\boldsymbol{\vec{\tau} = \vec{r} \times \vec{F}}$
The magnitude of torque τ is given by, $\boldsymbol{\tau = rF\sin\theta = F(r\sin\theta)}$

Mention the SI unit and dimensions of torque.
SI unit of torque is *Nm (newton-metre).*Dimensions are ML^2T^{-2}

How do you determine the direction of torque?
The direction of torque is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$ and can be determined by right hand rule.

Mention the two factors on which torque of a rotating body depends.
(i) Magnitude of the force.
(ii) Perpendicular distance of the point of application of the force from the axis of rotation.

Discuss the maximum and minimum values of torque.
If $\theta = 0$ the force acts in the direction of position vector, then $\tau = rF\sin 0 = 0$
If $\theta = 90^0$ Force acts perpendicular to the position vector then $\tau = rF\sin 0 = 90^0 = rF$

Define angular momentum (moment of momentum).
Angular momentum of a particle about an axis of rotation is defined as the vector product of linear momentum of the particle and the perpendicular distance of the particle from the axis of rotation.

Give the expression for angular momentum.
The angular momentum is given by, $\boldsymbol{\vec{l} = \vec{r} \times \vec{p}}$
The magnitude of the angular momentum vector is, $\boldsymbol{l = r\,p\sin\theta}$ where θ is the angle between $\vec{r}$ and $\vec{p}$

Mention the SI unit and dimensions of angular momentum.

SI unit of angular momentum is *kgm²s⁻¹* and dimensions are ML^2T^{-1}

Derive the relation between torque and angular momentum of a particle.

We know, $\vec{l} = \vec{r} \times \vec{p}$

Differentiating both sides with respect to t we have $\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$

$$\frac{d\vec{l}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$\frac{d\vec{l}}{dt} = \vec{r} \times \vec{F} + \vec{v} \times (m\vec{v})$$

$$\frac{d\vec{l}}{dt} = \vec{r} \times \vec{F} + m(\vec{v} \times \vec{v})$$

$$\frac{d\vec{l}}{dt} = \vec{r} \times \vec{F} \qquad \because (\vec{v} \times \vec{v} = 0)$$

$$\boldsymbol{\frac{d\vec{l}}{dt} = \vec{\tau}}$$

Thus, the time rate of change of angular momentum of a particle is equal to the torque acting on it.

Derive the relation between torque and angular momentum for a system of particles.

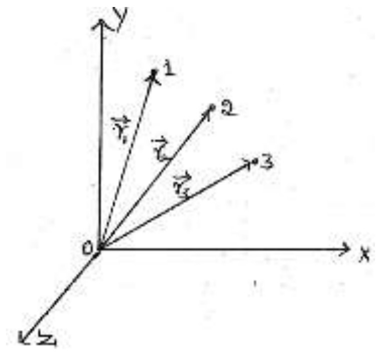
Consider a system of n-particles.

Let $\vec{l}_1, \vec{l}_2, \vec{l}_3 \ldots\ldots \vec{l}_n$ be the angular moments of the particles of the system respectively about the origin O.

The angular momentum of the system of particles is given by,

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \cdots + \vec{l}_n$$

$$\vec{L} = \sum_{i=1}^{n} \vec{l}_i$$



Differentiating with respect to t, $\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{n} \vec{l}_i\right)$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \frac{d\vec{l}_i}{dt}$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{\tau}_i)_{net}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{int} + \vec{\tau}_{ext}$$

We have separated the contribution of the external and internal torques to the total (net) torque.

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{r}_i \times \left(\vec{F}_{int}\right)_i + \sum_{i=1}^{n} \vec{r}_i \times \left(\vec{F}_{ext}\right)_i$$

The contribution of internal force to the total torque on the system is zero, because the forces between any two particles of the system are equal and opposite, and these forces are directed along the line joining the two particles.

$$\sum_{i=1}^{n} \left(\vec{F}_{int}\right)_i = 0 \Rightarrow \vec{\tau}_{int} = 0 \quad \text{and} \quad \tau_{net} = \tau_{ext}$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} (\vec{\tau}_i)_{ext}$$

$$\boldsymbol{\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}}$$

Thus, the time rate of change of angular momentum of a system of particles is equal to the net external torque acting on the system.

State and explain law of conservation of angular momentum for a system of particles.

If the total external torque on the system of particles is zero, the total angular momentum of the system of particles does not change with time.

Explanation: We have $\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$

If $\vec{\tau}_{ext} = 0$, then $\frac{d\vec{L}}{dt} = 0$ and $\vec{L} = Constant$

This is the law of conservation of angular momentum.

Equilibrium of a rigid body

What is equilibrium of a rigid body?

A rigid body is said to be in ***mechanical equilibrium***, if both its linear momentum and angular momentum are not changing with time.

Give the general conditions of equilibrium of a rigid body.

(i) If the total force on the body is zero then the total linear momentum of the body does not change with time. This is the condition for ***translational equilibrium***.

We have $\frac{d\vec{p}}{dt} = \sum_{i=1}^{n} \vec{F}_i$

If $\sum_{i=1}^{n} \vec{F}_i = 0$, then $\frac{d\vec{p}}{dt} = 0$ and $\vec{p}$ is constant

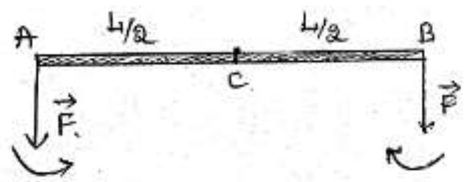
(ii) If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time. This is the condition for ***rotational equilibrium*** of the body.

We have $\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{\tau}_i$

If $\sum_{i=1}^{n} \vec{\tau}_i = 0$, then $\frac{d\vec{L}}{dt} = 0$ and $\vec{L}$ is constant.

What is meant by partial equilibrium? Explain with example.

A body may be in translational equilibrium and not in rotational equilibrium or it may be in rotational equilibrium and not in translational equilibrium. Then the body is said to be in partial equilibrium.



Ex: (i) consider a rod AB of negligible mass and length L.

Let two equal and parallel force acts on the two ends of the rod as shown.

Since the forces are parallel, net force on the rod is $\sum \vec{F} = \vec{F} = 2\vec{F} \neq 0$

Hence the rod is *not* in the translational equilibrium.

But the torque at A is, $\tau_1 = F \times \frac{L}{2}$ which tends to rotate the rod anticlockwise.

The torque at B is, $\tau_2 = F \times \frac{L}{2}$ which tends to rotates the rod clockwise.

Hence the net torque on the rod is zero. So the rod is *in* rotational equilibrium.

(ii) Now the force at B in the figure is reversed.

Now we have same rod with two equal and opposite force applied perpendicular to the rod.

Now the torque at A is, $\tau_1 = F \times \frac{L}{2}$ in anticlockwise direction.

at B, $\tau_2 = F \times \frac{L}{2}$ in clockwise direction.

Net torque is $\tau_1 + \tau_2 \neq 0$

But Force at A is exactly equal force at B and in opposite direction.

So net force is $= F + (-F) = 0.$

Here the rod is *in* translational equilibrium but *not* in rotational equilibrium.

What is couple? Give examples.

Two equal and opposite forces with different lines of action is known as couple.

Ex: (i) A tap is opened or closed when our figures apply a couple on it.

(ii) A lid of a bottle is also opened and closed when our fingers apply a couple on it.

Keep in mind

* When a couple acts on a body, the body is in translational equilibrium but not in rotational equilibrium. Thus a couple rotates the body.

What is a lever? Give example.

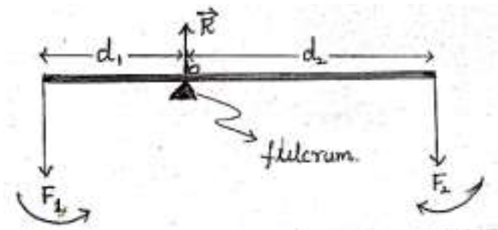
An ideal lever is a light rod pivoted at a point along its length. It works on the principle of moments. **Ex:** A see- saw on the children's playground.

Explain the principles of moments for a lever.

Consider two forces F_1 and F_2 parallel each other and perpendicular to the rod.

Let these forces act on the rod at distances d_1 and d_2 respectively from the fulcrum.

Let $\vec{R}$ be the reaction of the support at fulcrum which is directed opposite to the forces F_1 and F_2.



For rotational equilibrium, $d_1F_1 + (-d_2F_2) = 0$

$$d_1F_1 - d_2F_2 = 0$$

$$\boldsymbol{d_1F_1 = d_2F_2}$$

Anticlockwise moment of force = clockwise moment of force.

This is known as the principle of moments.

In the case of lever Force F_1 is known as load and force F_2 is known as effort.

Distance from the fulcrum d_1 is called load arm. Distance d_2 is called as effort arm

$$\textbf{Load arm} \times \textbf{load} = \textbf{effort arm} \times \textbf{effort}$$

The above equation expresses the principle of moments for a lever.

What is mechanical Advantage?

In the equation, $\frac{F_1}{F_2} = \frac{d_2}{d_1}$, the ratio (F_1/F_2) is called the mechanical advantage (MA) of the lever.

$$\boldsymbol{MA = \frac{F_1}{F_2} = \frac{d_2}{d_1}}$$

When will the mechanical advantage of a lever is greater than one and what does it mean?

If d_2 is larger than the d_1, then MA is greater than one. MA is greater than one means; small effort is enough to lift a large load.

Centre of gravity

Define centre of gravity.

Centre of gravity (G) of a body is defined as the point where the whole weight (Gravitational force) of the body is supposed to act.

When will centre of gravity of the body coincide with the centre of mass? Explain.

Centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity free space.

When a body is balanced, the body is in translational and rotational equilibrium.

If $\vec{r}_i$ is the position vector of the i^{th} particle of the body with respect to the centre of gravity, then the torque about centre of gravity due to force of gravity is zero.

i.e $\tau_g = \sum \tau_i = \sum \vec{r}_i \times m_i \vec{g} = 0$

If $\vec{g}$ is same for all particles, $\vec{g} \sum m_i \vec{r}_i = 0$

$$\sum m_i \vec{r}_i = 0 \qquad (since\ g \neq 0)$$

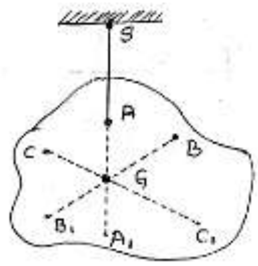
as $m_i \neq 0$, $\vec{r}_i = 0$

Thus Centre of gravity of the body coincides with the centre of mass.

Keep in mind

- ∗ If the body is extended that $\vec{g}$ varies from part to part of the body, then the cenre of gravity and centre of mass will not coincide.
- ∗ Basically centre of mass and centre of gravity are two different concepts. Centre of mass depends only on the distribution of mass of the body.

How do you determine the centre of gravity of a rigid body?



(i) Consider a body of irregular shape suspend the body from the points A,B,C …… respectively.

(ii) Now make lines AA_1, BB_1, CC_1, ………..

(iii) The point where all these lines intersect is the position of the centre of gravity of the body.

Moment of Inertia

What is moment of inertia (Rotational inertia)?

The property of the body by virtue of which it opposes or resists changing its state of rotational motion is called rotational inertia or moment of inertia.

Starting from the kinetic energy of a rigid body obtain an expression for moment of inertia of the body.

Consider a rigid body of mass *M* consisting of n-particles.

Let the body is rotating with angular velocity ω about the given axis of rotation.

The total kinetic energy of the body is the sum of the kinetic energies of the entire particle constituting the body.

$$K=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+.\dots\dots.+\frac{1}{2}m_nv_n^2$$

$$K=\frac{1}{2}\ m_1(r_1\omega)^2+\frac{1}{2}m_2(r_2\omega)^2+.\dots\dots\dots+\frac{1}{2}\ m_n\ (r_n\omega)^2$$

$$K\ =\frac{1}{2}\ \omega^2[m_1r_2^2+m_2r_2^2+.\dots\dots+m_nr_n^2]$$

$$K=\frac{1}{2}\omega^2\sum_{i=1}^{n}m_ir_1^2 \qquad ----(1)$$

$$We\ have,\ \ K=\frac{1}{2}I\omega^2 \qquad ----(2)$$

From Equation (1) and (2)

$$\boldsymbol{I=\sum_{i=1}^{n}m_ir_i^2}$$ I is called moment of Inertia.

Mention the SI unit and dimensions of moment of inertia.

Its unit in SI system is *kgm*2. Dimensions are ML^2T^0

Compare the moment of inertia of a body in rotational motion with mass of a body in translational motion.

In translational motion, greater in the mass of the body, greater is the force required to produce the linear acceleration in it. Thus in translational motion mass of the body is a measure of its inertia.

In rotational motion a torque is applied to produce angular acceleration. Moment of inertia is a measure of rotational inertia of the body. In rotation the moment of Inertia plays a similar role as mass does in the translational motion.

What is a fly wheel? Where do you find the use of fly wheel?

A fly wheel is a circular disc, whose most of the mass is concentrated on its rim and it rotates about an axel passing through its centre and perpendicular to its plane.

The machines such as steam engine and automobile engine that produce rotational motion use fly wheel.

Why a fly wheel is used in an engine of a train (vehicle)? Explain.

Jerky motion of a vehicle can be prevented by attaching a fly wheel with its engine.

The *fly wheel has large moment of inertia*. Whenever there is a sudden increase or decrease in the speed of vehicle, the fly wheel opposes this sudden increase or decrease in the speed of vehicle. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for passengers on the vehicle.

Mention the factors on which moment of inertia depends.

Moment of inertia depends on,

(i) Position of the axis of rotation with respect to the body.

(ii) Orientation of the axis of rotation.

(iii) Mass of the rotating body.

(iv) Distance from the axis of rotation to the distribution of mass.

What is radius of gyration?

It the distance of a mass point from the axis of rotation, whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the whole body about the axis.

Obtain the expression for radius of gyration from the definition of moment of inertia.

The moment of inertia of the body about the axis of rotation is given by, $I = \sum_{i=1}^{n} m_i r_i^2$

If m is the mass of the each particle, then, $I = \sum_{i=1}^{n} m\, r_i^2$

$$I = m \sum_{i=1}^{n} r_i^2$$

$$I = m(r_1^2 + r_2^2 + r_3^2 + \dots\dots\dots + r_n^2)$$

If the body contains n particles, then by multiplying and dividing RHS by n,

$$I = mn\frac{(r_1^2 + r_2^2 + \dots\dots\dots + r_n^2)}{n}$$

$$I = M\left(\frac{r_1^2 + r_2^2 + r_3^2 \dots\dots\dots + r_n^2}{n}\right)$$

$$\boldsymbol{I = Mk^2}$$

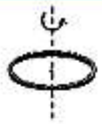
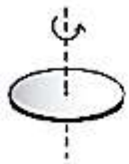
k is called radius of gyration and is given by, $\boldsymbol{k = \left[\frac{r_1^2 + r_2^2 + \dots\dots\dots + r_n^2}{n}\right]^{\frac{1}{2}}}$

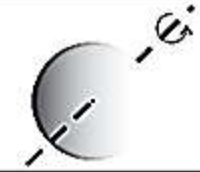
Mention the SI unit of radius of gyration.

SI unit of radius of gyration is *metre(m).*

Mention the expressions for moments of inertia of some regular bodies about specific axis.

Moments of Inertia of some regular shaped bodies about specific axes

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius R	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$MR^2/2$
(5)	Circular disc, radius R	Diameter		$MR^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		MR^2
(7)	Solid cylinder, radius R	Axis of cylinder		$MR^2/2$

(8)	Solid sphere, radius R	Diameter		$2MR^2/5$

Dynamics of rotational motion

Mention the kinematic equations for rotational motion about a fixed axis.

They are, $\omega = \omega_0 + \alpha t$, $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ and $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Obtain the expression for angular momentum of body in case of rotation about of fixed axis in terms of angular velocity and moment of inertia.

Consider a rigid body of mass M rotating with an angular velocity $\vec{\omega}$ along z-axis.

The rigid body is made up of large number of elements.

Consider one such element of mass m_i whose position vector is $\vec{r}_i$ and linear momentum is $\vec{p}_i$.

The angular momentum of this element about the axis of rotation is given by,

$$\vec{l}_i = \vec{r}_i \times \vec{p}_i = (rp\sin 90^0)\hat{k}$$
$$\vec{l}_i = (r_i p_i)\hat{k} = (r_i m_i v_i)\hat{k}$$
$$\vec{l}_i = (r_i m_i r_i \omega)\hat{k} = \left(m_i r_i^2 \omega\right)\hat{k}$$

The total angular momentum is given by,

$$\vec{L} = \sum \left(m_i r_i^2 \omega\right)\hat{k}$$
$$\vec{L} = \left(\omega\hat{k}\right)\sum m_i r_i^2$$
$$\vec{L} = \left(\omega\hat{k}\right)I$$
$$\boldsymbol{\vec{L} = (I\omega)\hat{k}} \text{ or } \boldsymbol{L = I\omega}$$

State and explain law of conservation of angular momentum in terms of angular velocity and moment of inertia.

In the absence of external torque, the net angular momentum of the system is conserved.

Explanation: We have $\vec{L} = I\vec{\omega}$

Differentiating both sides with respect to t

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(I\vec{\omega})$$

But $\frac{d\vec{L}}{dt} = \tau_{net}$

$$\therefore \frac{d}{dt}(I\vec{\omega}) = \tau_{net}$$

If the net external torque acting on that body is zero then,

$$\frac{d}{dt}(I\omega) = 0$$

$\boldsymbol{I\omega = constant}$ This is the law of conservation of angular momentum.

Illustrate law of conservation of angular momentum with exmples.

1. Suppose a man is sitting on a rotating table with his arms stretched outward. When the man with draws his arms towards his chest, the moment of inertia of the man decreases. Hence his angular speed increases.

2. A ballet dancer varies her angular speed by stretching her legs and arms out or in. As she stretches her legs and arms out, her moment of inertia increases and angular speed decreases.
3. When a diver Jumps from the spring board he curls his body by rolling his arms and legs in, by doing so he decreases his moment of inertia and hence angular speed increases.
4. When a planet revolving around the sun in an elliptical orbit comes near the sun, its speed increases. This is because as the planet comes near the sun, its moment of inertia decreases and hence its angular velocity increases.

Compare translational (Linear) and rotational motion.

Linear motion	Rotational motion about a Fixed axis
1. Displacement, x	Angular displacement, $\Delta\theta$
2. Velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
3. Acceleration, $a = \frac{dv}{dt}$	Angular acceleration, $\alpha = \frac{d\omega}{dt}$
4. Mass, M	Moment of Inertia, I
5. Force, F = Ma	Torque, $\tau = I\alpha$
6. Work, $W = Fds$	work, $W = \tau d\theta$
7. Kinetic energy, $K = \frac{1}{2}\, mv^2$	Kinetic energy, $K = \frac{I\omega^2}{2}$
8. Power, $P = Fv$	Power, $P = \tau\omega$
9. Linear momentum, $p = mv$	Angular momentum, $L = I\omega$

CHAPTER 7 GRAVITATION

Who proposed geocentric theory? Give the brief account of the theory.

Ptolemy about 2000 years ago proposed geocentric model. According to this theory, the sun, the moon and all planets, were in a uniform motion in circles called epicycles with the *motionless earth at the centre*.

Who proposed heliocentric theory? Give the brief account of the theory.

In 15 century Nicolas Copernicus proposed a definitive model, the helio-centric theory, according to which the earth and all other planets move in a *circular orbit* around the sun.

Keep in mind

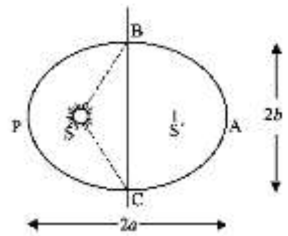
A more elegant model in which the sun was the centre around which the planet revolved was mentioned by Aryabhatta in 5 century AD in his treatise.

Kepler's Laws of planetary motion

State and explain Kepler's laws of planetary motion.

(1) Law of orbits: All planets move in *elliptical* orbits with the sun situated at one of the foci of the ellipse.

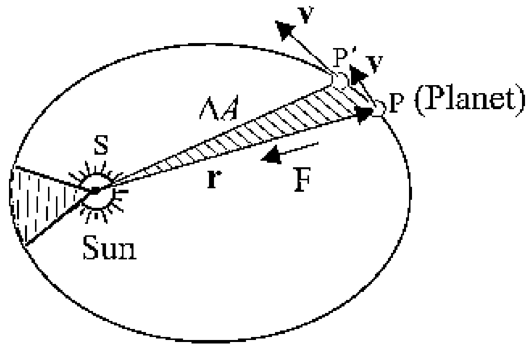
Explanation: S' and S are the foci, a is semi major axis and b is the semi minor axis.



(2) Law of areas: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

Explanation: Let the sun be at one of the foci of the ellipse. Then the area sweep out by the planet of mass m in time Δt is,

$$\frac{\Delta A}{\Delta t} = \textbf{Constant}$$



(3) Law of periods: The Square of the time period of revolution of a planet is proportional to the cube of the semi major axis of the ellipse traced out by the planet.

Explanation: If T is the time period and a is semei major axis, then $T^2 \propto a^3$.

Which physical quantity is conserved in the case of law of areas?

Angular momentum is conserved in the case of law of areas.

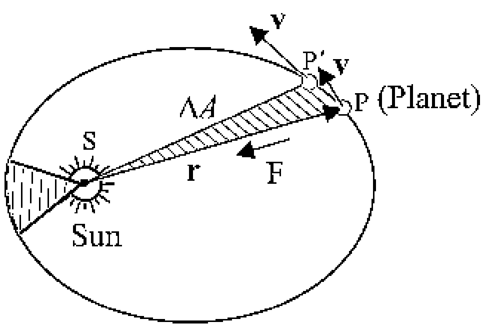
Show that the law of areas follows from the law of conservation of angular momentum.

Let the sun be at one of the foci of the ellipse.

Let the position and momentum of the planet be $\vec{r}$ and $\vec{p}$ respectively.

Then the area sweep out by the planet of mass m in time Δt is,

$$\Delta A = \frac{1}{2}(\vec{r} \times \vec{v}\Delta t)$$



$$\frac{\Delta A}{\Delta t} = \frac{1}{2}(\vec{r} \times \vec{v})$$
$$\frac{\Delta A}{\Delta t} = \frac{1}{2}\left(\vec{r} \times \frac{\vec{p}}{m}\right)$$
$$\frac{\Delta A}{\Delta t} = \frac{1}{2m}(\vec{r} \times \vec{p})$$
$$\frac{\Delta A}{\Delta t} = \frac{1}{2m}\vec{L} \; ----(1)$$

Now we have, $\frac{d\vec{L}}{dt} = \vec{\tau}$

But $\vec{\tau} = \vec{r} \times \vec{F}$ where $\vec{F}$ is Gravitation Force

But the force $\vec{F}$ is central force, i e. $\vec{F}$ is along $\vec{r}$

$\therefore \vec{r} \times \vec{F} = rF \sin 0 = 0$

Then $\frac{d\vec{L}}{dt} = 0$

$\vec{L}$ = constant

Using the above statement in equation (1)

$$\boldsymbol{\frac{\Delta A}{\Delta t} = \text{Constant}}$$

Newton's Universal law of Gravitation

What is Gravitational force?

Gravitational force is the force of attraction between the two bodies due to their masses. It is one of the basic forces of nature and is always attractive.

What is gravitation?

The tendency of bodies to move toward each other is called gravitation.

What is gravity?

The attractive force between earth and any other body is called gravity.

State Newton's Universal law of Gravitation and express in mathematical form.

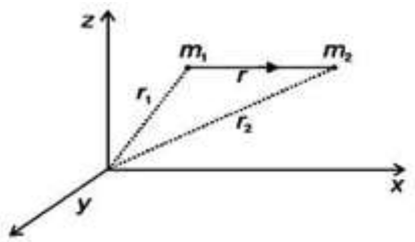
Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. If m_1 and m_2 are the masses of two bodies respectively and are separated by a distance r then,

$$|\vec{F}| \propto \frac{m_1 m_2}{r^2}$$
$$\boldsymbol{|\vec{F}| = G\frac{m_1 m_2}{r^2}}$$

where G is universal gravitational constant.

Express Newton's Universal law of Gravitation in vector form.

The force $\vec{F}$ is acting on a point mass m_2 due to another point mass m_1, and the force is directed towards point mass m_1.

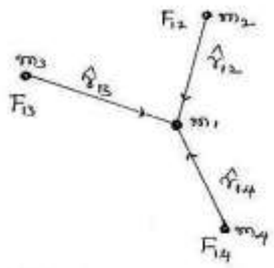


$$\vec{F}_{21} = \frac{G\, m_1 m_2}{|\vec{r}|^2}(-\hat{r})$$
$$\boldsymbol{\vec{F}_{21} = -\frac{G\, m_1 m_2}{|\vec{r}|^2}(\hat{r})}$$

where $\hat{r}$ is the unit vector from m_1 to m_2 and $\vec{r} = \vec{r_2} - \vec{r_1}$

Derive the expression for gravitational force due to multiple point masses.

If we have a collection of point masses the force on any one of them is the vector sum of the gravitational force exerted by the other point masses.

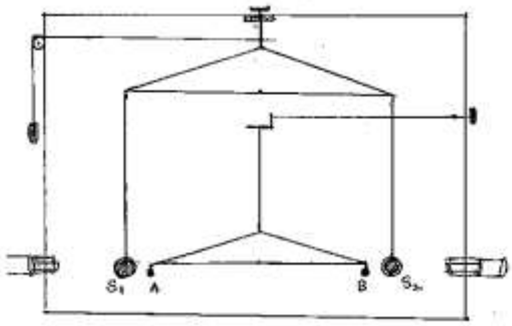


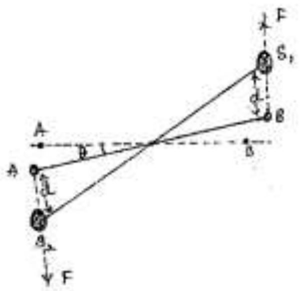
The total force on m_1 is, $F_1 = \frac{G\, m_2 m_1}{r_{21}^2}\ \hat{r}_{21} + \frac{G\, m_3 m_1}{r_{31}^2}\ \hat{r}_{31} + \frac{G\, m_4 m_1}{r_{41}^2}\hat{r}_{41}$

In case of gravitational force on a particle from a real (extended) object, we will divide the extended object in to deferential parts each of mass dm and each producing deferential force $\vec{F}$ on the particle,

in this limit sum becomes integral and $\vec{F}_1 = \int d\vec{F}$.

Describe the Henry Cavendish experiment to determine the value of gravitational constant.





In 1798 Henry Cavendish determined the value of G. The experimental arrangement is as shown. The Bar AB has two small lead sphere attached at it ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres S_1 and S_2 are brought close to the small ones but on opposite sides. The big sphere attracts the nearby small ones by equal and opposite forces. There is no net force on the bar but only torque which is equal to the F times the length of the bar and F is the force of attraction between a big and its neighboring small sphere. Due to this torque the suspended wire gets twisted such that the restoring torque of the wire equals to the gravitational torque.

If θ is the angle of twist, then restoring torque is equal to $\tau\theta$.

$$\therefore \frac{GMm}{d^2} L = \tau\theta$$

where $M \rightarrow$ mass of big sphere, $m \rightarrow$ mass of small spheres, $L \rightarrow$ length of the bar AB.

$$G = \frac{\tau\theta d^2}{MmL}$$

The measurement of G has been refined and the currently accepted value is,

$$G = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$$

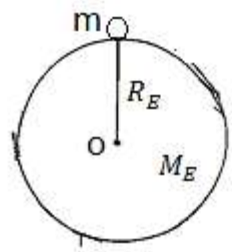
Keep in mind

The measurement of G by Cavendish experiment and with the knowledge of g and R_E, the mass of the earth M_E can be estimated. Because of this reason there is a popular statement regarding Cavendish, "Cavendish weighed the earth" .

Acceleration due to gravity

What is acceleration due to gravity?

The acceleration experienced by a body due to gravitational force of the earth is known as acceleration due to gravity.



Derive the expression for acceleration due to gravity.

Let us assume that the earth is uniform sphere of mass M_E.

Consider a body of mass m lying on the surface of the earth.

The magnitude of gravitational force acting on the body is given by,

$$F = \frac{GM_E m}{R_E^2}$$

The acceleration experienced by the body of mass m due to gravity is given by

$$g = \frac{F}{m} = \frac{GM_E m}{R_E^2 m}$$

$$\boldsymbol{g = \frac{GM_E}{R_E^2}}$$

Mention the factors on which acceleration due to gravity depends.

The acceleration due to gravity depends on (i) mass of the earth and (ii) Radius of the earth

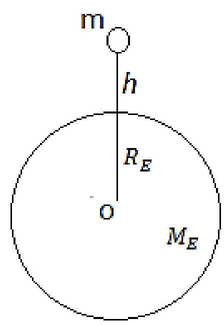
Obtain the expression for acceleration due to gravity above the surface of earth.

The acceleration due to gravity on the surface of the earth is given by, $g = \frac{GM_E}{R_E^2}$

Now consider a body at a height h above the surface of earth.

The acceleration due to gravity at height h is given by,

$$g_h = \frac{GM_E}{(R_E + h)^2}$$



by taking $\frac{g_h}{g}$, we have, $\frac{g_h}{g} = \frac{GM_E}{(R_E + h)^2} \times \frac{R_E^2}{GM_E}$

$$\frac{g_h}{g} = \frac{R_E^2}{(R_E + h)^2}$$

$$\frac{g_h}{g} = \frac{R_E^2}{R_E^2\left(1 + \frac{h}{R_E}\right)^2} = \frac{1}{\left(1 + \frac{h}{R_E}\right)^2}$$

$$\boldsymbol{g_h = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2}}$$

This shows that the acceleration due to gravity *decreases* as we go away from the surface of earth.

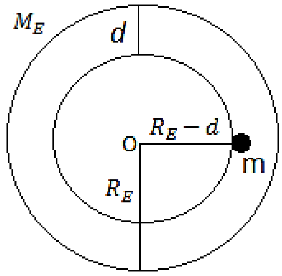
Further: $g_h = g\left(1 + \frac{h}{R_E}\right)^{-2}$

Using binomial theorem, $\left(1 + \frac{h}{R_E}\right)^{-2} \approx \left(1 - \frac{2h}{R_E}\right)$

As $h \ll R$, higher powers of $\frac{h}{R}$ can be neglected.

$$\boldsymbol{g_h = g\left(1 - \frac{2h}{R_E}\right)}$$

Obtain the expression for acceleration due to gravity below the surface of the earth.



The value of g on the surface of the earth is given by, $g = \dfrac{GM_E}{R_E^2}$

Now M_E = volume × density

$M_E = \dfrac{4}{3}\pi R_E^3 \rho$

Now, $g = \dfrac{G\left(\frac{4}{3}\pi R_E^3 \rho\right)}{R_E^3}$

$g = \dfrac{4}{3}\pi G R_E \rho$

When the body of mass m is taken to a depth d, the mass of the earth of radius $(R_E - d)$ will only be effective for the gravitational pull.

The outward shell will have no resultant effect on the mass of the body.

The acceleration due to gravity on the surface of the earth of radius $(R_E - d)$ is given by,

$g_d = \dfrac{4}{3}\pi G(R_E - d)\rho$

By taking $\dfrac{g_d}{g} = \dfrac{\frac{4}{3}\pi G(R_E - d)\rho}{\frac{4}{3}\pi G\, R_E \rho}$

$\dfrac{g_d}{g} = \dfrac{R_E - d}{R_E}$

$\dfrac{g_d}{g} = \dfrac{R_E\left(1 - \frac{d}{R_E}\right)}{R_E} = \left(1 - \dfrac{d}{R_E}\right)$

$\boldsymbol{g_d = g\left(1 - \dfrac{d}{R_E}\right)}$

As we go down below earth's surface the value of g *decreases* by a factor $\left(1 - \frac{d}{R_E}\right)$

Keep in mind

* The value of g is independent of mass of the body
* The weight of a body at a place on the surface of the earth is given by $W = mg$. since the value of g is maximum on the surface, the object weighs more on the surface.
* When $d = R$, $g_d = g\left(1 - \frac{R}{R}\right) = 0$, At the centre of the earth the value of g is zero.
  The value of g is more on the surface of the earth.
* The weight of a body at a place on the surface of the earth is given by $W = mg$. Since the value of g is maximum at poles than at the equator, the weight of the body is more at the poles than at the equator.

Gravitational potential energy and Earths satellites

What is gravitational potential energy?

Gravitational potential energy of a body is the work done in displacing the body from infinity to that point in the gravitational field. **OR**

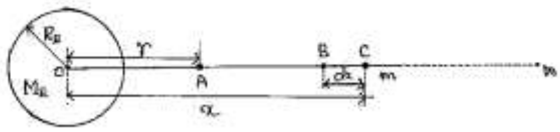
Potential energy of the body arising due to gravitational force is called gravitational potential energy.

Obtain the expression for Gravitational potential energy of a particle due to earth.

Consider a body of mass m placed at a distance x from the earth of mass M_E.

The gravitational force of attraction between the body and earth is given by ,

$$F = \frac{GM_E m}{x^2}$$



Now let the body of mass m be displaced from point C to B through a distance dx towards the earth, then Work done, $dW = Fdx$

$$dW = \frac{GM_E m}{x^2} dx$$

The total work done in displacing the body of mass m from infinity to a distance r towards the earth can be calculated by integrating the above equation between the limits $x = \infty$ to $x = r$.

$$\int dW = \int_{x=\infty}^{x=r} \frac{GM_E m}{x^2} dx$$

$$W = GM_E m \int_{\infty}^{r} \frac{1}{x^2}\, dx$$

$$W = GM_E m \int_{\infty}^{r} x^{-2} dx = GM_E m \left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$W = -GM_E m \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$W = -\frac{GM_E m}{r}$$

The work done is equal to the gravitation potential energy of the body and it is represented by V.

$$\boldsymbol{V = -\frac{GM_E m}{r}}$$

What is gravitation potential? Mention the expression for it.

The gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point.

$$V = -\frac{GM_E}{r} \times 1 \qquad (\because m = 1kg)$$

$$\boldsymbol{V = -\frac{GM_E}{r}}$$

Mention the SI unit and dimensions of gravitational potential.

The unit of gravitational potential is $J\,kg^{-1}$ and dimensions are $M^0L^2T^{-2}$

What is escape speed?

The minimum initial speed required for an object to escape from the earth's gravitational field (to reach infinity) is called escape speed.

Obtain the expression for escape speed of a body on the earth.

Consider an object of mass m is thrown upward so that it can reach infinity, then the speed there was v_f.

The energy of an object is the sum of Potential Energy and Kinetic energy.

At infinity, $E_\infty = \frac{1}{2} mv_f^2$ …………………….(1) (Potential energy = 0 at infinity)

Initially if the object was thrown with a speed v_i from point at a distance $(R_E + h)$ from the centre of the earth, the energy is given by,

$$E_i = \frac{1}{2}\, mv_i^2 - \frac{GmM_E}{(R_E + h)} \;\ldots\ldots\ldots (2)$$

By the principle of conservation of energy, equation (1) and (2) are equal.

$$\frac{1}{2}\, mv_i^2 - \frac{GM_E m}{(R_E + h)} = \frac{1}{2}\, mv_f^2$$

The RHS of the above equation is positive quantity with a minimum value zero, hence so must be the LHS.

$$\frac{1}{2}\, mv_i^2 - \frac{GM_E m}{(R_E + h)} \geq 0$$

$$\frac{1}{2}\, m\left(v_i^2\right)_{min} - \frac{GM_E m}{(R_E + h)} = 0$$

$$\frac{1}{2}\, m\left(v_i^2\right)_{min} = \frac{GM_E m}{(R_E + h)}$$

$$\frac{1}{2}\left(v_i^2\right)_{min} = \frac{GM_E}{R_E + h}$$

$$\left(v_i^2\right)_{min} = \frac{2GM_E}{R_E + h}$$

$$(v_i)_{min} = \sqrt{\frac{2GM_E}{R_E + h}}$$

If the object is thrown from the surface of the earth then $h = 0$

$$(v_i)_{min} = \sqrt{\frac{2GM_E}{R_E}}$$

But $\dfrac{GM_E}{R_E^2} = g,$ then $\dfrac{GM_E}{R_E} = gR_E$

$$\boldsymbol{(v_i)_{min} = \sqrt{2gR_E}}$$

What is the value of escape speed of a body (i) on the surface of earth and (ii) on the surface of moon?

(i) $(v_i)_{min} = 11.2\ km/s$ for the earth and (ii) $(v_i)_{min}$ for moon is 2.3 km/s

---

Keep in mind

* Escape speed of a body on the surface of moon is 2.3 km/s. Gas molecules if formed on the surface on the moon having velocities larger than this will escape from the gravitational pull of the moon. Because of this reason moon has no atmosphere.

What is a satellite?

Satellites are the celestial objects revolving around the planet.

What is an earth's satellite?

Earth satellite is an object which revolves around earth.

Which is the natural satellite of earth?

Earth has only one natural satellite - moon with a time period 27.3 days and its rotational period about it axis is also same as that of the time period.

What is orbital velocity?

The velocity required to put a satellite into its orbit around the earth is called orbital velocity.

Obtain the expression for orbital speed of a satellite around the earth.

Consider a satellite of mass m and speed v_0 in a circular orbit at a distance $(R_E + h)$ from the centre of the earth.

The centripetal force required for this obit is, $F_c = \frac{mv_0^2}{(R_E + h)}$

The centripetal force is provided by the gravitational force, $F = \frac{GmM_E}{(R_E + h)^2}$

Equating the equations, we get $\frac{mv_0^2}{(R_E + h)} = \frac{GmM_E}{(R_E + h)^2}$

$$v_0^2 = \frac{GM_E}{R_E + h}$$

$$\boldsymbol{v_0 = \sqrt{\frac{GM_E}{R_E + h}}}$$

For $h = 0$ (since $h << R_E$), we have $v_0 = \sqrt{\frac{GM_E}{R_E}}$

$$\boldsymbol{v_0 = \sqrt{gR_E}} \qquad \left(\because g = \frac{GM_E}{R_E^2}\right)$$

Give the relation between escape and orbital speed.

$(v_i)_{min} = \sqrt{2}\ v_o$

Obtain the expression for time period of a satellite around the earth.

In every orbit the satellite travels a distance $2\pi(R_E + h)$ with speed v_0, then its time period is,

$$T = \frac{2\pi(R_E + h)}{v_0} = \frac{2\pi(R_E + h)}{\left(\sqrt{\frac{GM_E}{(R_E + h)}}\right)}$$

$$T = \frac{2\pi(R_E + h)(R_E + h)^{\frac{1}{2}}}{\sqrt{GM_E}}$$

$$\boldsymbol{T = \frac{2\pi(R_E + h)^{\frac{3}{2}}}{\sqrt{GM_E}}}$$

Assuming the expression for time period arrive at Kepler's law of periods.

$$T = \frac{2\pi(R_E + h)^{\frac{3}{2}}}{\sqrt{GM_E}}$$

Squaring on both sides

$$T^2 = \frac{4\pi^2(R_E + h)^3}{GM_E}$$

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)(R_E + h)^3$$

$\boldsymbol{T^2 = K(R_E + h)^3}$ where $K = \frac{4\pi^2}{GM_E}$, Which is the Kepler's law of periods

Obtain the expression for total energy of the satellite.

We have, $E = V + KE$

$$E = -\frac{GM_E m}{(R_E + h)} + \frac{1}{2}\ mv_0^2$$

$$E = -\frac{GM_E m}{(R_E + h)} + \frac{1}{2} m \frac{GM_E}{(R_E + h)}$$

$$E = \frac{GM_E m}{(R_E + h)} \left(-1 + \frac{1}{2}\right)$$

$$\boldsymbol{E = -\frac{GM_E m}{2(R_E + h)}}$$

The total energy of an orbiting satellite is negative.

Keep in mind

* If total energy of an orbiting satellite is equal or greater than zero then the satellite does not remain in the orbit, it escapes from the earths pull.
* Negative energy implies that the satellite is bound to the earth.

CHAPTER 8 MECHANICAL PROPERTIES OF SOLIDS

Elasticity

What is elasticity of a body?

The property of a body, due to which it tends to regain its original size and shape when deforming force is removed, is called elasticity. The deformation caused is called as elastic deformation.

What is plasticity?

The property of a body due to which it does not regain its original size and shape when the deforming force is removed is called plasticity. The substances are called as plastics.

Stress and Strain

What is Stress?

The restoring force per unit area is known as stress. **OR** Stress can be defined as deforming force per unit area.

Stress $= \frac{\boldsymbol{F}}{\boldsymbol{A}}$

Mention the SI unit and dimensions of stress.

Its SI unit is Nm^{-2} or $pascal$ (Pa). Its dimensions are $ML^{-1}T^{-2}$

What is Strain?

The ratio of Change in configuration to the original configuration of the body is called strain.

Strain $= \frac{\boldsymbol{change\ in\ configuration}}{\boldsymbol{original\ configuration}}$

Strain has no unit. It is dimensionless quantity. It is a ratio.

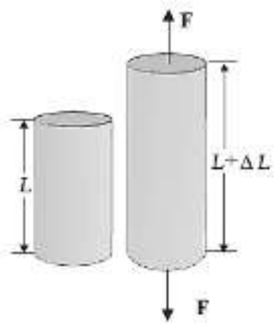
Mention the types of Stress and strain. There are three types in each.

Stress	Strain
(1) Normal stress (σ)	(1) Longitudinal strain (ε)
(2) Tangential stress / Shearing stress (σ_S)	(2) Shearing strain (θ)
(3) Hydraulic stress / Volume stress / Bulk stress (P)	(3) Volume strain

What is Normal stress($\boldsymbol{\sigma}$)? Mention its types.

It is defined as the restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types.

(i) Tensile stress (ii) Compressive stress



What is tensile stress?

When two equal and opposite forces are applied at the ends of a circular rod to increase its length, a restoring force normal to the cross-sectional area of the rod is developed. This restoring force per unit area of cross-section is known as tensile stress.

What is compressive stress?

When two equal and opposite forces are applied at the ends of a circular rod to decrease its length, a restoring force normal to the cross-sectional area of the rod is developed. This restoring force per unit area of cross-section is known as compressive stress.

Illustration:

A steel rod of area of cross section $3.14 \times 10^{-4} m^2$ is stretched by a force of $100\ kN$. Calculate the stress acting on the rod.

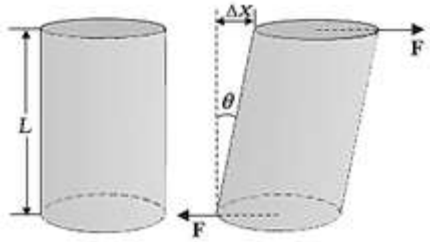
$$Normal\ stress = \frac{Force}{area} = \frac{100 \times 10^3}{3.14 \times 10^{-4}} = 3.18 \times 10^8\ Nm^{-2}$$

What is longitudinal strain?

It is defined as the ratio of the change in length to the original length of the body.

$$Longitudinal\ strain = \frac{Change\ in\ length}{Original\ length}$$

$$\boldsymbol{\varepsilon = \frac{\Delta L}{L}}$$



What is tangential stress / shearing stress($\boldsymbol{\sigma_S}$)?

The restoring force per unit area developed due to the applied tangential force is known as tangential stress or Shearing stress.

What is shearing strain($\boldsymbol{\theta}$)?

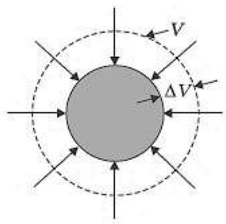
It is defined as the angle through which the face of the body originally perpendicular to the fixed face is turned when it is under shearing stress.

$$\text{Shearing strain} = \frac{\Delta x}{L}$$

$$\tan\theta = \frac{\Delta x}{L}$$

If θ is small, $tan\ \theta \approx \theta$ then, $\boldsymbol{\theta = \frac{\Delta x}{L}}$

What is hydraulic stress / volume stress / bulk stress($\boldsymbol{p}$)?



When an object is immersed in a fluid, the fluid exerts force on the surfaces of the object as a result the volume of the object decreases and the object is under stress known as hydraulic stress.

During hydraulic stress there is no change in the geometrical shape of the object.

What is volume strain?

It is defined as the ratio of change in volume to the original volume.

$$\boldsymbol{Volume\ strain = -\frac{\Delta V}{V}}$$

Negative sign indicates that volume decreases when the body is under bulk stress.

Hooke's law of elasticity

State and explain Hooke's law of elasticity.

For small deformations, the stress and strain are proportional to each other.

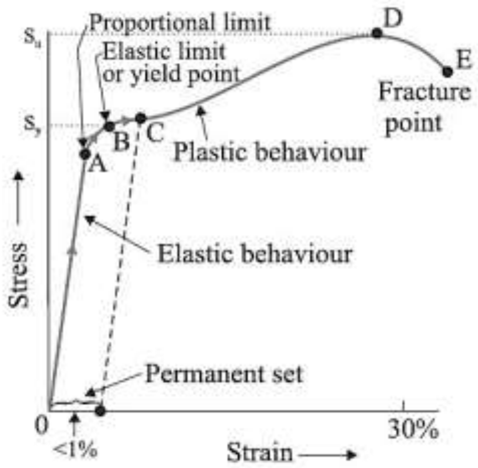
$$Stress \propto Strain$$

$$Stress = k\ (Strain)$$

The constant k is known as modulus of elasticity.

Mention the SI unit and dimensions of modulus of elasticity.
SI unit is Nm^{-2} and dimensions are $ML^{-1}T^{-2}$

Draw typical stress – strain graph for a metal and explain the important features in it.



In the region between O and A, the curve is linear and Hooke's law is obeyed. The body regains its original dimensions and the body behaves as an elastic body.

In the region from A to B, stress and strain are not proportional, but the body still regains its original dimensions after the removal of load. The point B is called Yield point (elastic limit) and the corresponding stress is called Yield strength (S_y).

If the stress is increased beyond B, the strain increases rapidly. This is represented by the region between B and D. When the load is removed, say at C, the body does not regain its original dimensions. The material is said to have a permanent set. The deformation is said to be plastic deformation.

The point D on graph is the ultimate tensile strength(S_u) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at E.
If the ultimate strength and fracture points are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

Distinguish between ductile material and brittle material.

Ductile material	Brittle material
The material showing large amount of plastic deformation between the elastic limit and the fracture point is called ductile material.	The material showing small amount of plastic deformation between the elastic limit and The fracture point is called brittle material.
They have permanent stretch without breaking	They fractured soon after the elastic limit is crossed
The ultimate strength and fracture points are far apart.	The ultimate strength and fracture points are close.

Know the terms

(a) Elastic limit or yield point: The maximum stress below which Hooke's law is applicable is called elastic limit.
(b) Yield strength: The stress corresponding to Yield point (elastic limit) called Yield strength.
(c) Permanent set: When a wire is stretched more, then it has permanent strain even when the stress is zero. Then wire is said to have permanent set.
(d) Plastic deformation: When a wire is stretched too much, then it has permanent strain even when the stress is zero. This behaviour of the material is called plastic deformation
(e) Ultimate tensile strength: The stress required to cause actual fracture of the material is called ultimate tensile strength.
(f) Fracture point: The stretched wire breaks for certain applied stress is called fracture point.

What are elastomers? Give examples.

The materials having large elastic region, but does not obey Hooke's law and have no well-defined plastic region are called elastomers. Ex: Rubber, tissue of aorta etc.

Modulus of Elasticity

What is modulus of Elasticity? Mention the types.

The ratio of stress and strain is called modulus of elasticity.

Types of modulus of elasticity: There are three types

(i) Young's modulus (Y)

(ii) Shear modulus / Rigidity modulus (G)

(iii) Bulk modulus (B)

Define Young's modulus($\boldsymbol{Y}$) of elasticity?

The ratio of Normal (tensile or compressive) stress to the longitudinal strain is defined as young's modulus.

Write the expression for magnitude of the Young`s modulus.

$$Y = \frac{Normal\ stress}{longitudinal\ strain}$$

$$\boldsymbol{Y = \frac{\left(F/_A\right)}{\left(\Delta L/_L\right)} = \frac{FL}{A\Delta L}}$$

Illustration:

Which is more elastic among – steel, copper and aluminium? Why?

Larger the value of young's modulus of the material, more elastic it would be. For steel, $= 200 \times 10^9\ Nm^{-2}$. For copper, $Y = 110 \times 10^9\ Nm^{-2}$. For aluminium $Y = 70 \times 10^9\ Nm^{-2}$.

Steel more elastic because its young's modulus is more than copper and aluminium.

Define Shearing modulus/Rigidity modulus($\boldsymbol{G}$).

The ratio of shearing stress to the corresponding shearing strain is called shear modulus.

Write the expression for magnitude of the Shearing modulus/Rigidity modulus.

$$G = \frac{Shearing\ stress}{Shearing\ strain}$$

$$\boldsymbol{G = \frac{\left(F/_A\right)}{\left(\Delta x/_L\right)} = \frac{FL}{A\Delta x}}$$

Define Bulk modulus($\boldsymbol{B}$)?

The ratio of hydraulic stress to the corresponding hydraulic strain (Volume strain) is called bulk modulus.

Write the expression for magnitude of the Bulk modulus.

$$\boldsymbol{B = \frac{Hydraullic\ stress}{Hydraullic\ strain} = -\frac{p}{\left(\Delta V/_V\right)} = -\frac{pV}{\Delta V}}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (V).

Illustration:

Compute the fractional change in the volume of glass sphere when subjected to a hydraulic pressure of $1.013 \times 10^{6} Nm^{-2}$. Given bulk modulus of glass is $3.7 \times 10^{10}\ Nm^{-2}$.

$$B = \frac{p}{\left(\Delta V/_{V}\right)}$$

$$\left(\Delta V/_{V}\right) = \frac{B}{p} = \frac{1.013 \times 10^{6}}{3.7 \times 10^{10}} = 2.74 \times 10^{-5}$$

What is compressibility(k)?

The reciprocal of the bulk modulus is called Compressibility **OR** It is defined as the fractional change in volume per unit increase in pressure.

$$k = \frac{1}{B} = \frac{1}{-\frac{p}{\left(\Delta V/_{V}\right)}}$$

$$\boldsymbol{k = -\frac{\Delta V}{pV}}$$

Keep in mind

- The SI unit of young's modulus, shear modulus and bulk modulus is Nm^{-2}.
- Generally for metals Young's moduli are large. Hence they are more elastic in nature.
- For a perfectly rigid body the young's modulus is infinite.
- It can be seen that Shear modulus is generally less than Young's modulus for most materials and $\approx \frac{Y}{3}$.
- Bulk modulus for a perfect rigid body and ideal gas is infinite.
- A solid has all types of moduli of elasticity but fluids have only bulk modulus of elasticity.
- Solids are least compressible than liquids. Gases are more compressible than liquids and solids.

Define lateral strain.

The ratio of change in diameter to the original diameter is called lateral strain.

Keep in mind

Within elastic limit this lateral strain is directly proportional to the longitudinal strain.

$$Lateral\ strain \propto Longitudinal\ strain$$

Define Poison's ratio(σ).

The ratio of the lateral strain to the longitudinal strain is called *Poison's ratio*.

Mention the expression for Poison's ratio.

$$Poison's\ ratio\ = \frac{Lateral\ strain}{Longitudinal\ strain}$$

$$\boldsymbol{Poison's\ ratio} = \frac{\left(\Delta d/_{d}\right)}{\left(\Delta L/_{L}\right)} = \frac{\boldsymbol{L\,\Delta d}}{\boldsymbol{d\,\Delta L}}$$

Why Poisons ratio has no unit and dimensions?

It has no dimensions and unit because Poison's ratio is a ratio of two strains and it is a pure number. The practical value of Poison's ratio lies between 0 and 0.5.

What is elastic Potential energy?

When a wire is put under a tensile stress, work is done against the inter-atomic forces. This work is stored in the wire in the form of Elastic potential energy.

Derive the expression for Elastic Potential energy.

Consider a wire of length L and area of cross-section A.

Let a force F be applied to stretch the wire.

If l be the length through which the wire is stretched.

Then, $\frac{F}{A} = Y \times \frac{l}{L}$

$F = \frac{YAl}{L}$

If the wire is stretched through a length dl, work done is given by, $dW = Fdl$

$dW = \frac{YAl}{L}dl$

Total work done to stretch the wire from 0 to l is, $W = \int dW = \int_0^l \frac{YAl}{L}dl$

$W = \frac{YAl^2}{2L}$

$W = \frac{1}{2}Y\left(\frac{l}{L}\right)^2 AL$

$W = \frac{1}{2} \times \frac{stress}{strain} \times (strain)^2 \times AL$

$W = \frac{1}{2} \times stress \times strain \times volume\ of\ the\ wire$

$$\boldsymbol{U = \frac{1}{2} \times stress \times strain \times volume\ of\ the\ wire}$$

Applications of Elastic behaviour

Explain the applications of Elastic behaviour of materials.

(1) In selecting metallic rope for crane.

Explanation: $Stress = \frac{Load}{Area} = \frac{Mg}{\pi r^2}$ where $r \to$ radius of the rope required

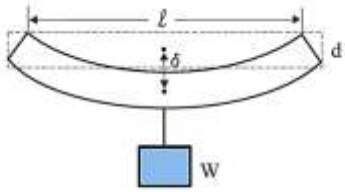
Let us take the lifted mass $M = 10^5 kg$ and $g = 10ms^{-2}$. The elastic limit of the steel is $30 \times 10^7 Nm^2$, Then the maximum stress on the rope is $30 \times 10^7 Nm^2$.

Radius, $r = \left(\frac{10^5 \times 10}{3.14 \times 30 \times 10^7}\right)^{\frac{1}{2}} = 0.0325m = 3.25cm$

The radius of the steel rope to lift $10^5 kg$ should be about $3cm$. Practically it will be a rigid rod but the rope is made up of a large number of thin wires braided together to make it flexible.

(2) In minimizing of the bending of loaded beam

Explanation: When a beam of length (l), breadth (b) and depth (d) loaded weight is as shown.

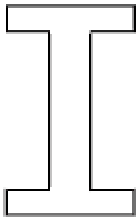


The depression of the beam is, $\delta = \dfrac{Mgl^3}{4bd^3Y}$

$$\delta \propto \frac{1}{d^3}$$

The depression in the loaded beam can be decreased effectively by increasing d.

In building and bridge construction, Iron girders used are not rectangular shape but in the shape of letter I. The I Shaped girder is much stronger than the rectangular shaped girder.



(3) To estimate the maximum height of the mountain.

Explanation: The stress due to all the material on the top of the mountain should be less than the critical shearing stress at which the rocks flow.

If h is the height of the mountain and ρ be the density of the rocks of the mountain, then the pressure at the base of the mountain$=\rho gh = stress$.

The elastic limit of a typical rock$=3 \times 10^8 Nm^{-2}$

The stress must be less than the elastic limit; otherwise the rock begins to sink under its own weight.

$$\rho gh < 3 \times 10^8$$

$$h < \frac{3 \times 10^8}{\rho g} = \frac{3 \times 10^8}{3 \times 10^3 \times 10}$$

$$h < 10^4 m$$

$$h \approx 10km$$

It may be noted that the height of the Mount Everest is nearly $9km$.

Chapter 9 **MECHANICAL PROPERTIES OF FLUIDS**

Know the terms and concepts

What are fluids?

The materials that can flow are called fluids. Liquid and gases are collectively known as fluids. Unlike a solid a fluid has no definite shape of its own.

What is hydrostatics (Fluids statics)?

The study of fluids at rest is known as hydrostatics.

What is hydrodynamics (Fluids dynamics)?

The study of fluids in motion is termed as hydrodynamics.

Define density of a substance.

Density of a substance is defined as the mass per unit volume of the substance.

$$\boldsymbol{\rho = \frac{M}{V}}$$

Mention the SI unit and dimensions of density.

SI unit of density is kgm^{-3} and dimensions are ML^{-3}

Define relative density.

It is defined as the ratio of the density of the substance to the density of water.

$$Relative\ density = \frac{Density\ of\ the\ substance}{Density\ of\ the\ water}$$

Mention the SI unit of relative density.

Relative density has no unit. It is a pure number.

What is the density of water at 4^{0}C?

Density of water at 4^0C is maximum and is equal to $1000kgm^{-3}$

Pressure

Define Pressure.

The pressure is defined as the magnitude of the force acting perpendicular to the surface of an object per unit area of the object.

$$\boldsymbol{P = \frac{F}{A}}$$

Mention the SI unit and dimensions of pressure.

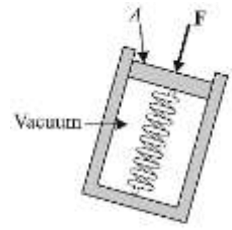
SI unit of pressure is Nm^{-2} or $pascal(Pa)$. Dimensions of pressure is $ML^{-1}T^{-2}$

Keep in mind

Pressure is a scalar quantity, because hydrostatic pressure is transmitted equally in all directions, when force is applied, which shows that a definite direction is not associated with pressure.

Describe the measurement of pressure.

The normal force exerted by the fluid at a point may be measured and the arrangement is as shown. It consists of an evacuated chamber with a spring that is calibrated to measure the force acting on the piston. This device is placed at a point inside the fluid. The inward force exerted by the fluid on the piston is balanced by the outward spring force and is thereby measured.



State Pascal's Law.

The pressure in a fluid at rest is same at all points if they are at the same height.

Show that pressure exerted is same in all directions in a fluid at rest OR Explain Pascal's law.

Consider an element $ABC\text{-}DEH$ in the form of a right-angled prism in the fluid.

As the element is very small, every part of it is located at the same height from the liquid surface. Then the effect of gravity is same at all these points.

Let F_a, F_b and F_c be the normal forces exerted by the fluid on the faces $BEHC, ADHC$ and $ADEB$ respectively.

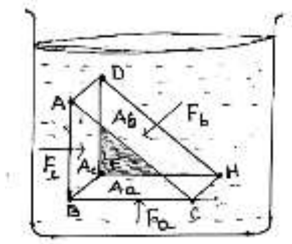
Let A_a, A_b and A_c be the area of the faces $BEHC, ADHC$ and $ADEB$ respectively.

Since the element $ABC\text{-}DEH$ is in equilibrium, net force acting on that element should be zero.

$$F_c = F_b \sin\theta$$
$$F_a = F_b \cos\theta$$

By geometry, $A_c = A_b \sin\theta$

$$A_a = A_b \cos\theta$$



Pressure on $ADEB = \dfrac{F_c}{A_c} = \dfrac{F_b \sin\theta}{A_b \sin\theta} = \dfrac{F_b}{A_b}$ $\quad --- (1)$

Pressure on $BEHC = \dfrac{F_a}{A_a} = \dfrac{F_b \cos\theta}{A_b \cos\theta} = \dfrac{F_b}{A_b}$ $\quad --- (2)$

Pressure on $ADHC = \dfrac{F_b}{A_b}$ $\quad --- (3)$

The above equations says, $\dfrac{F_c}{A_c} = \dfrac{F_a}{A_a} = \dfrac{F_b}{A_b}$ implies that $P_a = P_b = P_c$

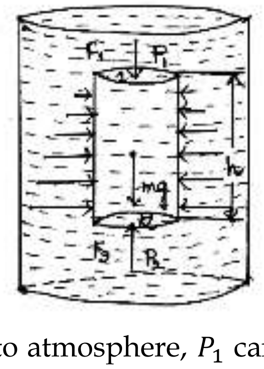
Hence, pressure exerted is same in all directions in a fluid at rest.

Derive the expression for pressure inside a liquid at rest.

Consider a vessel containing a liquid of density ρ, which is in equilibrium.

Consider a cylindrical element of fluid having area of base A and height h.

As the fluid is in equilibrium, the net force acting on it is zero.



$$F_2 = F_1 + mg$$

But $P = \dfrac{F}{A}$ therefore $F_1 = P_1 A$ and $F_2 = P_2 A$

$$P_2 A = P_1 A + (\rho V)g$$
$$P_2 A = P_1 A + \rho A h g$$
$$P_2 = P_1 + \rho g h$$

If the *point* 1 is in the figure is shifted to the top of the fluid, which is open to atmosphere, P_1 can be replaced by atmospheric pressure P_a and P_2 by P then,

$$\boldsymbol{P = P_a + \rho g h}$$

What is Gauge Pressure? Mention the expression.

Gauge pressure is the difference of the actual pressure and the atmospheric pressure at a point.

$$\boldsymbol{P - P_a = \rho g h}$$

What is atmospheric pressure?

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.

Mention the value of atmospheric pressure at sea level.

At sea level it is 1 $atmospheric\ pressure$ which is equal to $1.013 \times 10^5 Pa$.

Illustration:

Find the pressure exerted below a column of water, open to atmosphere, at depth (a) $5\ m$ (b) $20\ m$ (Given: $\rho = 1000\ kg\ m^{-3},\ g = 10\ ms^{-2},\ P_a = 1.013 \times 10^5\ Pa$

$P = P_a + \rho g h$

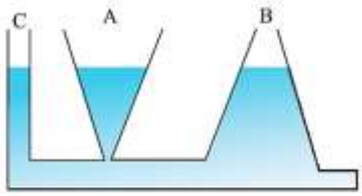
(a) $P = 1.013 \times 10^5 + 1000 \times 10 \times 5 = 1.013 \times 10^5 + 50000 = 1.013 \times 10^5 + 0.5 \times 10^5$

$P = 1.513 \times 10^5\ Pa$

(b) $P = 1.013 \times 10^5 + 1000 \times 10 \times 20 = 1.013 \times 10^5 + 200000 = 1.013 \times 10^5 + 2 \times 10^5$

$P = 3.013 \times 10^5\ Pa$

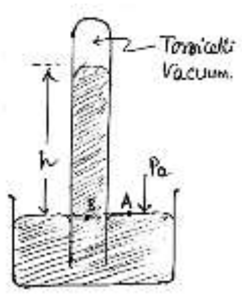
Explain Hydrostatic paradox.



Consider three vessels A, B and C of different shapes. They are connected at the bottom by a horizontal pipe. On filling with water, the level in the vessel is same. Though they hold different amount of water, any how the pressure of the water at the bottom is same. This result is known as hydrostatic paradox.

How do you measure the atmospheric pressure? Explain.

Torricelli invented a ***mercury barometer*** to measure the atmospheric pressure. It consists of a long glass tube closed at one end and filled with mercury and inverted in to a trough of mercury as shown. The space in the tube above the mercury column is almost empty and can be neglected. This space is called Torricelli space.



Pressure at A is atmospheric pressure = P_a

Pressure at B is $P = \rho g h$ where ρ is density of mercury.

From Pascal's Law, Pressure at A = Pressure at B

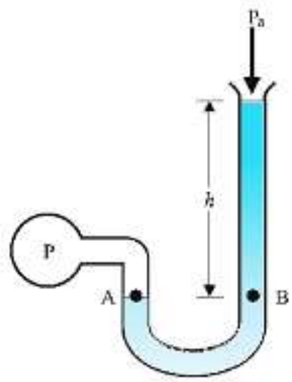
$\boldsymbol{P_a = \rho g h}$ where ρ is density of mercury.

Keep in mind

* At sea level, the mercury column in the barometer is found to have a height of $76cm$. The pressure equivalent to this column is 1 $atmospheric\ pressure$ ($1\ atm$).
* A common way of stating pressure is in terms of cm or $mm\ of\ Hg$.
* A pressure equivalent to $1mm$ is called a $torr$. $1mm\ of\ Hg = 1\ torr = 133Pa$
* $mm\ of\ Hg$ and $torr$ are used in medicine and physiology.
  In meteorology a common unit is the bar and $milli\ bar$. ($1bar = 10^5 Pa$)

How do you measure the pressure difference between a system and atmosphere?

Open tube monometer is an instrument for measuring pressure difference. It consists of a U-tube containing a low density liquid for measuring small pressure differences or a high density liquid for large pressure differences. One end of the tube is open to the atmosphere and the other end is connected to the system whose pressure is to be measured.



Pressure at $A = P$

Pressure at $B = P_a + \rho g h$ where ρ is density of liquid in the monometer.

From Pascal's Law, Pressure at A = Pressure at B

$$\boldsymbol{P = P_a + \rho g h}$$

$$\boldsymbol{P - P_a = \rho g h}$$

The gauge pressure is $P - P_a = \rho g h$ which is proportional to the height of the liquid.

State Pascal's law for transmission of fluid pressure.

Whenever external pressure is applied on any part of the fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

Mention the applications of the Pascal's law for transmission of fluid pressure.

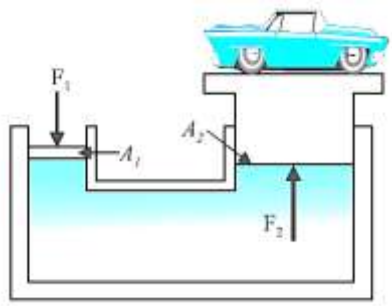
Hydraulic machines work based on the Pascal's law for transmission of fluid pressure. In these devices fluids are used for transmitting pressure.

What are hydraulic machines? Give examples.

The devices which work on the Pascal's law are known as hydraulic machines. **Ex:** Hydraulic lift, hydraulic brakes etc.

Explain the working of Hydraulic lift. <u>OR</u>
Explain how Pascal's law is applied in a hydraulic lift?

It consists of a horizontal fluid filled container. Both the ends of the container are fitted with two cylinders having pistons of different area of cross-section as shown in the figure.



Explanation: Let the force of magnitude F_1 be applied to a small piston of surface area A_1

This generates a pressure, $P = \frac{F_1}{A_1}$

This pressure is transmitted undiminished through the fluid to a larger piston of surface area A_2

Hence, $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$

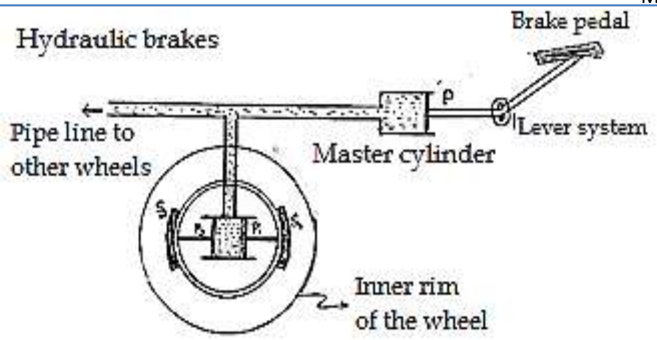
$$\boldsymbol{F_2 = F_1\left(\frac{A_2}{A_1}\right)}$$

As $A_2 > A_1$, $F_2 > F_1$

This shows that a small force applied on the smaller piston appears as a large force on the larger piston.

Explain the working hydraulic brakes?

When we apply a little force on the pedal with our foot, the master piston P moves inside the master cylinder and the pressure caused is transmitted through the brake oil to act on a piston of large area (P_1 and P_2). A large force acts on the piston and is pushed down expanding the brake shoes (S_1 and S_2) against the brake lining and retard the motion of the wheel.



State Archimedes' principle.
When a body is immersed completely or partially in a liquid it appears to lose a part of its weight and this apparent loss of weight is equal to the weight of the liquid displaced by the body.

Streamline (Steady) flow

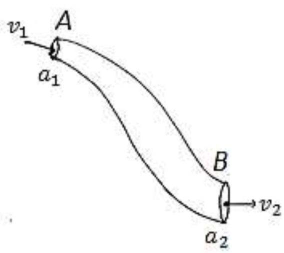
Mention the types of fluid flow.
The flow of fluids is divided into two types, namely (i) Streamline (Steady) flow (ii) Turbulent flow

What is Streamline (Steady) flow?
If a fluid flows such that the velocity of its particles at a given point remains constant with time, then the fluid is said to have streamline flow.

What is a Streamline?
The path followed by the particle of a fluid in a streamline flow or steady flow is called streamline.

Mention the properties of streamlines.
(i) The tangent at any point on the line of flow gives the direction of flow.
(ii) The streamline may curve and bend, but they cannot cross each other.

Mention and explain the equation of continuity.
Consider streamline flow of a fluid of density ρ, through a pipe AB of non-uniform cross-section.
Let v_1 be the velocity of the liquid entering at A of the cross-sectional area a_1 normal to the surface.
Let v_2 be the velocity with which it flows out at B where the area of cross-section a_2 normal to the surface.
Mass of fluid entering at A per second $= Density \times volume = \rho(a_1 v_1)$
Mass of fluid entering at B per second $= Density \times volume = \rho(a_2 v_2)$



Since the flow is steady,
The mass of the fluid entering per second is equal to the mass of the fluid flowing out per second.

$$\rho a_1 v_1 = \rho a_2 v_2$$
$$a_1 v_1 = a_2 v_2$$

This equation is called equation of continuity and it is the statement of law of conservation of mass in flow of incompressible fluids.
In general, $\boldsymbol{av = constant}$ This is known as equation of continuity.

What is Turbulent flow? Give examples.

When the speed of flow exceeds a limiting value called critical velocity, the orderly motion of the fluid is lost and it acquires an unsteady motion called turbulent motion.

Ex: Floods, hurricanes, whirlpools etc.

Mention the differences between streamline flow and turbulent flow

Streamline flow		Turbulent flow	
1	It is a regular and orderly flow	1	It is irregular and disorderly flow
2	The lines of flow are parallel to each other	2	The lines of flow are not parallel to each other
3	The velocity of the flow is less than the critical velocity	3	The velocity of the flow is greater than the critical velocity
4	Different particles cross a given point with same velocity	4	Different particles cross a given point with different velocity

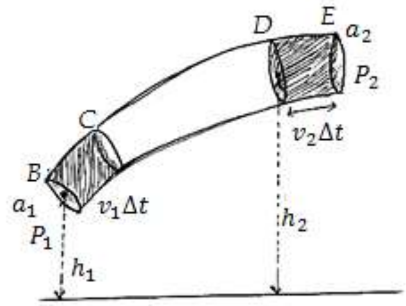
Bernoulli's principle

State and explain Bernoulli's principle.

For streamline flow of an ideal (non-viscous, incompressible) fluid, the sum of pressure, the kinetic energy per unit volume and potential energy per unit volume remains a constant.

Explanation: Consider an incompressible and non-viscous fluid flowing through a pipe BE of varying cross-sectional area.

Let the flow is streamline.



Let P_1 and P_2 be the pressure and a_1 and a_2 be the area of cross-section at B and D respectively.

Let v_1 is the speed of fluid at B and v_2 at D.

Then, $$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

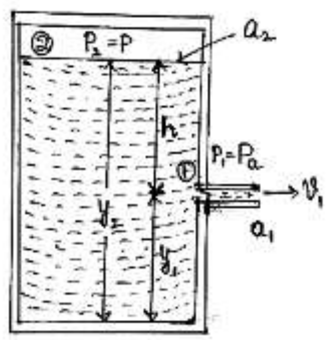
In general, $$\boldsymbol{P + \rho g h + \frac{1}{2}\rho v^2 = constant}$$

Mention the limitations of Bernoulli's theorem:

1) Bernoulli's equation applies only to non-viscous fluids.
2) Bernoulli's theorem cannot be applied to compressible fluids.
3) Bernoulli's equation does not hold good for non-steady or turbulent flow.

State and prove Torricelli's law (Speed of Efflux)

Torricelli discovered that, the expression for speed of efflux from an open tank is similar to that of a freely falling body.



Proof: Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom of the tank.

Let y_2 be the height of the free surface of the liquid from the bottom of the tank.

Let P be the pressure of air above the free surface of the liquid.

From the equation of continuity, $a_1 v_1 = a_2 v_2$

$$\boldsymbol{v_2 = \frac{a_1}{a_2} v_1}$$

Where $a_1 \rightarrow$ Cross-sectional area of the hole

$a_2 \rightarrow$ Cross-sectional area of the tank

$v_1 \rightarrow$Velocity of fluid coming out of the hole

$v_2 \rightarrow$Velocity of fluid at the top surface of the liquid

Since $a_2 \gg a_1$, top layer of the liquid is approximately at rest. i.e. $v_2 = 0$

Also the pressure of the fluid at the hole P_1 is same as that of the atmospheric pressure P_a.

Applying Bernoulli's equation to point 1 and 2,

$$P_a + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P + \rho g y_2$$

$$\frac{1}{2}\rho v_1^2 = P - P_a + \rho g y_2 - \rho g y_1$$

$$\frac{1}{2}\rho v_1^2 = P - P_a + \rho g(y_2 - y_1)$$

$$v_1^2 = \frac{2}{\rho}(P - P_a + \rho g h) \qquad \text{(since } y_2 - y_1 = h\text{)}$$

$$v_1^2 = \frac{2(P - P_a)}{\rho} + 2gh$$

$$v_1 = \sqrt{\frac{2(P - P_a)}{\rho} + 2gh}$$

If the tank is open to atmosphere then, $P = P_a$

$$\boldsymbol{v_1 = \sqrt{2gh}}$$

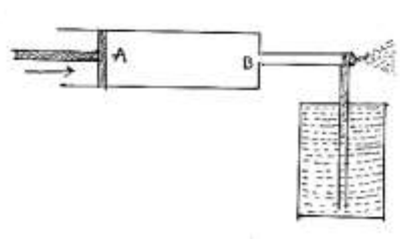
This is same as the speed of a freely falling body. This equation is known as Torricelli's law.

Name the instruments which work on Bernoulli's principle.

Filter pumps (Aspirators), carburettor.

Explain the working of Filter pumps (Aspirators).

When a fluid passes through a region at a large speed, the pressure in that region decreases. This fact is used in this device.



The air in the tube A is pushed using a piston. As the air passes through the constriction B its speed is considerably increases and consequently pressure drops. Thus the liquid rises from the vessel and is sprayed with the expelled air. Bunsen burner, atomiser and sprayers work on the same principle.

Keep in mind

Carburettor: The function of the carburettor is to deliver the rightly proportioned mixture of petrol vapour and air to the cylinder of a petrol engine. This also works based on Bernoulli's principle.

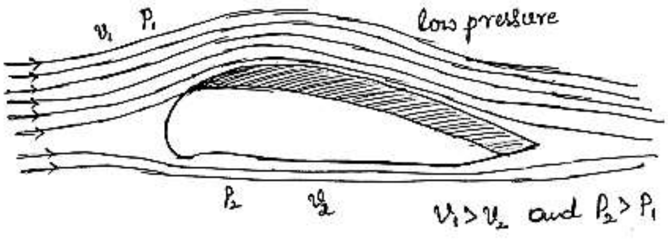
What is Dynamic lift?

Dynamic lift is the force that acts on a body by virtue of its motion through a fluid **<u>OR</u>** the difference in pressure causes an upwards thrust called dynamic lift

What is Magnus effect?

Dynamic lift due to spinning is called Magnus effect.

Explain the Uplift of an aircraft based on Bernoulli's principle.



The shape of the wings of aircraft is specially designed so that the velocity of the layers of air on its upper surface is more than that on the lower surface.

According to Bernoulli's principle where velocity of the fluid is high, the pressure is low and vice-versa. So the pressure P_1 is low at the upper surface of the wing and pressure P_2 is high at the lower surface of the wing. This difference in pressure causes an upwards thrust called dynamic lift on the wings of the air-craft.

Viscosity

What is Viscosity?

The property of a liquid by virtue of which it opposes relative motion between its different layers is called viscosity.

What is viscous force?

In case of a liquid having relative motion between the layers internal forces are developed, which retard the relative motion. These retarding forces are called viscous force.

Keep in mind

- ∗ The viscous force does not operate as long as the liquid is at rest. They come to play only when there is a relative motion between its layers.
- ∗ Greater viscosity favours streamline flow whereas lower viscosity causes turbulent motion.

Define Co-efficient of viscosity (η).

The coefficient of viscosity of a fluid is defined as the ratio of shearing stress to the strain rate.

Mention the expression for Co-efficient of viscosity.

The co − efficient of viscosity, $$\eta = \frac{F/A}{v/l} = \frac{Fl}{vA}$$

Mention the SI unit and dimensions of Co-efficient of viscosity.

The SI unit of viscosity is Nsm^{-2}. It can be expressed also in *pascal second*. The dimensions are $ML^{-1}T^{-1}$.

Explain the temperature dependence of viscosity.

1) As the temperature of the liquid increases, the distance between the molecule increases. Hence the magnitude of cohesion force decreases and the viscosity decreases.

2) When the temperature of the gas increases, the change of momentum and number of collisions also increases and hence the co-efficient of viscosity increases.

What is viscous drag or drag force?

When an object moves relative to a fluid, the fluid exerts a friction like retarding force on the object. This force is called viscous drag or drag force.

State and explain Stokes' law.

The viscous force acting on an object moving in a fluid is directly proportional to the velocity of the object.

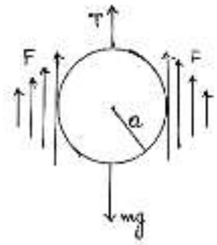
Viscous drag(F) acting on a spherical body of radius a moving with velocity v in a fluid of co-efficient of viscosity η is given by, $\boldsymbol{F = 6\pi\eta a v}$

What is terminal velocity (v_t)?

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity. This constant velocity is called terminal velocity.

Obtain the expression for Terminal velocity of an object <u>OR</u>

Show that terminal velocity of a sphere falling through a viscous medium is proportional to Square of its radius.



Consider a spherical body of radius a falling through a viscous fluid having density σ and co-efficient of viscosity η.

Let ρ be the density of the material of the body.

The viscous forces acting on that spherical body are,

(i) its weight (mg) in downward direction.

(ii) upward thrust T equal to the weight of the displaced fluid.

(iii) viscous drag F in a direction opposite to the direction of motion of the body.

Net downward force acting on that body $= mg - T - F$

When the body attains terminal velocity, acceleration, $a = 0$

From Newton's I law, Net force acting on the body is zero.

$$mg - T - F = 0$$

Now, (i) $mg = (V \times \rho)g = \frac{4}{3}\pi a^3 \rho g$

(ii) $T = Weight\ of\ the\ fluid\ displaced = (V \times \sigma)g = \frac{4}{3}\pi a^3 \sigma g$

(iii) According to Stocke's law, $F = 6\pi\eta a v_t$

Therefore, $\frac{4}{3}\pi a^3 \rho g - \frac{4}{3}\pi a^3 \sigma g - 6\pi\eta a v_t = 0$

$$\frac{4}{3}\pi a^3 g(\rho - \sigma) = 6\pi\eta a v_t$$

$$v_t = \frac{\frac{4}{3}\pi a^3 g(\rho - \sigma)}{6\pi\eta a}$$

$$\boldsymbol{v_t = \frac{2}{9}\frac{a^2 g(\rho - \sigma)}{\eta}}$$

Keep in mind

* $v_t \propto a^2$, Terminal velocity depends on square of the radius of the sphere.
* $v_t \propto \frac{1}{\eta}$, Terminal velocity depends inversely on viscosity of the medium.
* Falling of rain drops through air and the descent of a parachute can be explained using Stokes' law.

What is Reynolds number?

The number which gives an approximate idea about whether the flow would be turbulent or not is called Reynolds number, denoted by R_e.

Mention the expression for Reynolds number.

If η is the viscosity and ρ is the density of fluid flowing with a speed v in a pipe of diameter d, the value of R_e is given by,

$$\boldsymbol{R_e = \frac{\rho v d}{\eta}}$$

Classify the flow of fluid based on Reynolds number?

(i) For laminar or streamline flow, $R_e < 1000$

(ii) For the turbulent flow, $R_e > 2000$

(iii) For $1000 < R_e < 2000$, the flow becomes unsteady.

What is critical Reynolds number?

The value of Reynolds number at which the turbulence just occurs is called critical Reynolds number.

What is critical velocity?

The maximum velocity of a fluid in a tube for which the flow remains streamline is called Critical velocity.

Mention the importance of Reynolds number.

The value of Reynolds number is very much useful in designing of ships, submarines, race cars, aeroplanes etc.

Surface Tension

What is Surface Tension?

The property of a liquid at rest by virtue of which its free surface behaves like a stretched membrane under tension and tries to minimise the surface area is called surface tension. OR
Surface tension is defined as, force per unit length acting on the surface of the liquid.

Mention the SI unit and dimensions of surface tension.

Unit of surface tension is Nm^{-1} and dimensions are ML^0T^{-2}.

What is Surface energy?

The potential energy of the surface molecules per unit area of the surface is called surface energy.

$$\boldsymbol{Surface\ energy = \frac{potential\ energy}{area}}$$

Mention the SI unit and dimensions of surface energy.

Unit of surface energy is $joule/metre^2$ and dimensions are ML^0T^{-2}

Write the expression to Measure surface tension.

$$\boldsymbol{S = \frac{F}{2l}}$$

The quantity S is the magnitude of surface tension and it is equal to the surface energy per unit area of the liquid surface.

What is angle of contact?

The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is called as angle of contact. It is denoted by θ.

When a liquid does wet the surface of solid?

When angle of contact (θ) is acute angle($\theta < 90^0$), the molecules of the liquid are attracted strongly to those of solids and the liquid will wet the surface

Ex: (i) Water on glass.
(ii) Water on plastic sheet.

When a liquid does not wet the surface of solid?

When angle of contact (θ) is obtuse angle($\theta > 90^0$),, the molecules of the liquid are attracted strongly to themselves and weakly to those of solids and the liquid will not wet the surface.

Ex: (i) water-leaf interface.
(ii) water in waxy or oily surface.
(iii) Mercury on any surface. etc.

Why drop of a liquid and bubbles assumes the shape of a sphere OR Explain the formation of drops and bubbles.

Due to surface tension, surface of liquid always has a tendency to have least surface area. For a given volume a sphere has the minimum surface. Therefore a small volume of liquid take the shape of a sphere.

Mention the expression for excess pressure inside a liquid drop and inside a Bubble.

Excess pressure inside a liquid drop is,

$$(\boldsymbol{P_i - P_o}) = \frac{\boldsymbol{2S}}{\boldsymbol{r}}$$

Excess pressure inside a bubble is

$$(\boldsymbol{P_i - P_o}) = \frac{\boldsymbol{4S}}{\boldsymbol{r}}$$

Where $(P_i - P_o)$ is pressure difference between inside and outside and S is the surface energy.

What is capillarity?

When a capillary tube is dipped in water, the water rises up in the tube. This rise of liquid in a capillary tube is known as capillarity.

Mention the expression for capillary rise.

$$\boldsymbol{h} = \frac{\boldsymbol{2S\cos\theta}}{\boldsymbol{\rho g r}}$$

where ρ is the density of water and h is called capillary rise

Mention the practical applications of capillarity.

(i) The oil in the lamp rises in the wick to its top by capillary action.
(ii) Sap and water rise up to the top of the leaves of the tree by capillary action.
(iii) Ink is absorbed by the blotting paper due to capillarity.
(iv) The moisture rises in the capillaries of the soil to the surface, where it evaporates. To prevent this and preserve moisture in the soil, capillaries must be destroyed by ploughing and levelling fields.

Chapter 10 THERMAL PROPERTIES OF MATTER

Temperature and Heat

What is temperature?
Temperature is relative measure or indication of hotness or coldness of a body.

What is Heat?
Heat is the form of energy transferred between two systems or a system and its surroundings by virtue of temperature difference.

Distinguish between heat and temperature.

Heat	Temperature
Heat is the form of energy transferred between two systems.	It is the indication of hotness or coldness of a body
S I Unit of heat energy is joule (J).	The S I unit of temperature is kelvin (K).
It is the cause.	It is the effect.

Know the terms and concepts

Thermometry: The branch of science which deals with the measurement of temperature of a substance is known as thermometry.

Thermometer: A device used to measure the temperature of a body is called thermometer.

* Commonly used thermometers are liquid-in-glass type.
* Mercury and Alcohol are the liquids in most of this type of thermometer.
* Temperature of a body determines the direction of flow of energy.
* Heat energy flows from body of higher temperature to a body of lower temperature.
* Heat gained by a body is taken as positive while heat lost by a body is taken as negative.

Which principle is used in designing thermometer?
The principle of thermometer is, when a substance is heated, some of its physical properties change. The commonly used property is variation of the volume of a liquid with temperature.

Which is the convenient temperature points used in designing the thermometer?
The ice (freezing) point and steam (boiling) point of water are the two convenient fixed points.

Describe the different temperature scales used to measure the temperature.

(i) Celsius scale (^{0}C): Celsius scale of temperature was invented by Andres Celsius. In this scale, the melting point of pure ice at standard atmospheric pressure is 0^{0}C and marked as lower fixed point. The boiling point of pure water at standard atmospheric pressure is 100^{0}C and marked as upper fixed point. The interval between these two points is divided into 100 equal parts. Each part is taken as “One degree Celsius”.

(ii) Fahrenheit scale (^{0}F): Fahrenheit scale was invented by Gabriel Fahrenheit. In this scale, the melting point of pure ice at standard atmospheric pressure is 32^{0}F and marked as lower fixed point. The boiling point of pure water at standard atmospheric pressure is 212^{0}F and marked as the upper fixed point. The interval between these two points is divided into 180 equal parts and each part is known as "One degree Fahrenheit".

Kelvin's scale of temperature: This scale was suggested by Kelvin. Absolute zero is foundation of the Kelvin temperature scale. The zero of the absolute scale of temperature is denoted by $0\ K$ and known as absolute zero.

Establish the relation between Celsius and Fahrenheit scale.

A plot of Fahrenheit temperature (t_F) versus Celsius temperature (t_C) is as shown.

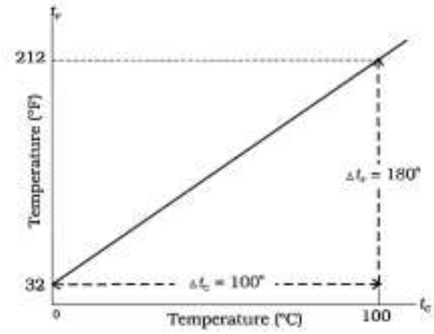
The equation of the straight line is given by,

$$y = mx + C$$

or
$$t_F = \frac{180}{100} t_c + 32$$

$$t_F - 32 = \frac{180}{100} t_c$$

$$\boldsymbol{\frac{t_F - 32}{180} = \frac{t_c}{100}}$$



Write the relation between Celsius and Kelvin scale.

$t_K = t_C + 273.15$

Write the values of ice point of water, boiling point of water and normal temperature of human body in different temperature scales.

Details	Celsius scale (°C)	Fahrenheit scale (°F)	Kelvin scale (K)
Ice point of water	0	32	273.15
Boiling point of water	100	212	373.15
normal temperature of human body	37	98.6	310.15

Illustration:

What is that temperature at which the

(i) Celsius and Fahrenheit scale give same temperature value?

(ii) Fahrenheit reading is double that of the Celsius reading?

(i) Put $t_F = t_c = x$

$$\frac{t_F - 32}{180} = \frac{t_c}{100} \Rightarrow \frac{x - 32}{180} = \frac{x}{100}$$

Solving we get, $x = -40$

So, at $-40^0 C$, Fahrenheit reading is also $-40^0\ F$

(ii) Put $t_F = x$ and $t_c = 2x$

$$\frac{2x - 32}{180} = \frac{x}{100} \Rightarrow \frac{2x - 32}{9} = \frac{x}{5}$$

Solving, $x = 160^0\ C$ and $2x = 320^0\ F$

What is the drawback of liquid-in-glass thermometers?

Liquid in glass thermometers shows different readings for the temperatures other than the fixed points. This is because of different expansion properties of liquids.

How do you rectify the drawback of liquid-in-glass thermometer?

This problem can be removed, if a thermometer uses a gas. They give same reading regardless of which gas is used and experiments show that that all gases at low densities exhibit same expansion behaviour.

What are Gas laws?

The laws which describe the behaviour of a gas at different conditions are called gas laws.

How do you explain the behaviour of a given quantity of gas?

The behaviour of a given quantity of gas is explained using the variables such as pressure, volume and temperature.

State and explain Boyle's law.

When temperature is held constant, the volume of a given mass of gas is inversely proportional to its pressure.

$$V \propto \frac{1}{P}$$

$$\boldsymbol{PV = constant}$$

State and explain Charles law.

At constant pressure, the volume of a given mass of an ideal gas is directly proportional to its absolute temperature.

$$V \propto T$$

$$\boldsymbol{\frac{V}{T} = constant}$$

Arrive at ideal gas equation using gas laws.

For low density gases, we can combine Boyle's law and Charles law into a single relationship as,

$$PV = constant$$

$$\frac{V}{T} = constant$$

$$\frac{PV}{T} = constant$$

$$\frac{PV}{T} = R \qquad \text{where } R \to constatnt$$

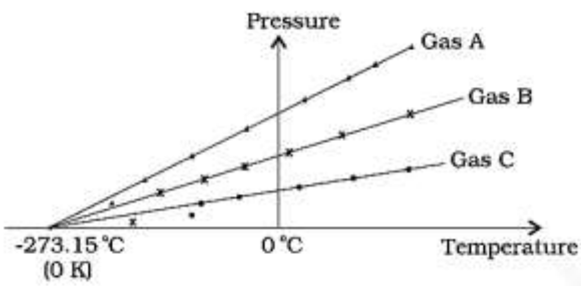
For μ moles of gas, $\frac{PV}{T} = \mu R$

$$\boldsymbol{PV = \mu RT}$$

This equation is called Ideal gas equation. Where R is a constant called universal gas constant and $R = 8.31\, J\, mol^{-1} K^{-1}$

Describe the absolute temperature scale <u>OR</u> Write a note on Kelvin scale of temperature.

At constant volume, P versus T graph is as shown. The relationship is linear over a large temperature range. It looks as pressure might reach zero, with decreasing temperature and gas remains to be a gas. Thus, pressure of a gas becomes zero at $-273.15\,^0C$. This limit which is the same no matter what kind of gas is used, is called absolute zero. The temperature scale based on this zero is called absolute temperature scale or kelvin's scale.



What is absolute zero?

The temperature at which the pressure of an ideal gas becomes zero is termed as absolute zero.

Effects of Heat

Mention the effects of Heat.

* Thermal expansion
* Rise in temperature
* Change in state

What is thermal expansion?

The increase in dimensions of a body due to increase in its temperature is called thermal expansion.

Mention the types of thermal expansion.

* Linear expansion
* Area expansion
* Volume expansion

What is linear expansion?

When a rod like solid is heated, its length increases. This expansion in length is called linear expansion.

Mention the factors on which the linear expansion depends? Explain.

The increase in length (Δl) is directly proportional to, (i) its original length (l_0), (ii) the change in temperature (ΔT) and the material of the solid.

Mathematically, $\Delta l \propto l_0 \Delta T$

$$\frac{\Delta l}{l_0} \propto \Delta T$$

$$\frac{\Delta l}{l_0} = \alpha_l \, \Delta T$$

Where $\alpha_l \rightarrow$ constant and called co-efficient of linear expansion.

Define Co-efficient of linear expansion. Mention its SI unit.

It is defined as the increase in length per unit length per degree increase in temperature.

S I unit of Co-efficient of linear expansion is K^{-1}.

What is thermal stress?

Consider a metallic rod whose ends are fixed rigidly. When the temperature of the rod increases, its length increases. Since there is no space left to increase its length, so it bends. If the rod is not allowed to bend, then it will be under a great stress. This is known as thermal stress.

Derive the expression for thermal stress developed in rod.

Let l be the length of the rod and A be its cross-sectional area.

α_l be the co-efficient of linear expansion of the material of the rod.

Let ΔT be the increase in temperature and Δl be the increase in its length.

Then, $\quad \dfrac{\Delta l}{l} = \alpha_l \, \Delta T$

But $\frac{\Delta l}{l} = longitudinal\ strain$

$strain = \alpha_l\ \Delta T$

Young's modulus, $Y = \frac{Thermal\ stress}{Longitudinal\ strain} = \frac{Thermal\ stress}{\alpha_l\ \Delta T}$

$\boldsymbol{Thermal\ stress = Y\ \alpha_l\ \Delta T}$

Derive the expression for force developed in the rod due to Thermal stress.

We have, $Thermal\ stress = \frac{Force\ due\ to\ stress}{Area\ of\ cross - section}$

Force, $F = Thermal\ stress \times Area\ of\ cross - section$

$F = Y\ \alpha_l\ \Delta T \times A$

$F = YA\ \alpha_l\ \Delta T$

$\boldsymbol{F = \frac{YA\ \Delta l}{l}}$

Illustration:

A person uses a steal measuring tape tape is exactly $50.000\ m$ long at a temperature of $20^0\ C$. What is its length on a hot summer day when the temperature is$35^0\ C$? ($\alpha_{steel} = 1.2 \times 10^{-5}\ K^{-1}$)

$\Delta T = 15^0\ C, l = 50.000\ m$

$\Delta l = l_0 \alpha_l \Delta T = 50.000 \times 1.2 \times 10^{-5} \times 15 = 9.0 \times 10^{-3}\ m = 0.009\ m$

Increase in length $= l_0 + \Delta l = 50.000\ m + 0.009\ m = 50.009\ m$

What is Area (Superficial) expansion?

When a solid planar body is heated, its area increases. This expansion in the area is called area expansion.

Mention the factors on which the area expansion depends? Explain.

The increases in area (ΔA) is directly proportional to, (i) its original area (A_0), (ii) its change in temperature (ΔT) and the material of the solid.

Mathematically, $\Delta A \propto A_0\ \Delta T$

$\frac{\Delta A}{A_0} \propto \Delta T$

$\frac{\Delta A}{A_0} = \alpha_A\ \Delta T$

Where $\alpha_A \rightarrow$ constant and called co-efficient of area expansion.

Define Co-efficient of area expansion. Mention its SI unit.

It is defined as the increase in area per unit area per degree increase in temperature.

S I unit of Co-efficient of area expansion is K^{-1}.

What is volume (Cubical) expansion?

When a substance is heated its volume increases. The expansion in the volume is called volume expansion.

Mention the factors on which the volume expansion depends? Explain.

The increases in volume (ΔV) is directly proportional to, (i) its original volume (V_0), (ii) its change in temperature (ΔT) and nature of solid.

Mention the factors on which the volume expansion depends? Explain.

Mathematically, $\Delta V \propto V_0 \, \Delta T$

$$\frac{\Delta V}{V_0} \propto \Delta T$$

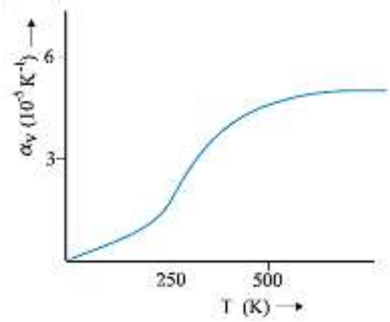
$$\frac{\Delta V}{V_0} = \alpha_V \, \Delta T$$

Where $\alpha_V \rightarrow$ constant and called co-efficient of volume expansion.

Define Co-efficient of volume expansion. Mention its SI unit.

It is defined as the increase in volume per unit volume per degree increase in temperature.

S I unit of Co-efficient of volume expansion is K^{-1}.



Explain the variation of Co-efficient of volume expansion (α_V) with temperature – For SOLIDS and LIQUIDS

At ordinary temperature Solids and Liquids expand less compared to the gases. For liquids, the co-efficient of volume expansion is relatively independent of the temperature.

Derive $\alpha = \frac{1}{T}$ for an ideal gas, where the symbols have their usual meaning **OR**

Show that Co-efficient of volume expansion (α_V) varies inversely with temperature for GASES.

The ideal gas equation is given by, $PV = \mu RT$

At constant pressure, the equation becomes, $P\Delta V = \mu R \, \Delta T$

Taking, $$\frac{P\Delta V}{PV} = \frac{\mu R \, \Delta T}{\mu RT}$$

we have, $$\frac{\Delta V}{V} = \frac{\Delta T}{T}$$

$$\frac{\Delta V}{V}\frac{1}{\Delta T} = \frac{1}{T}$$

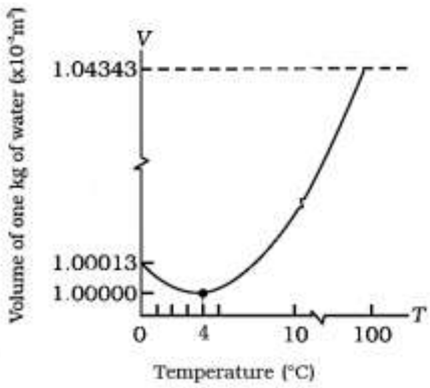
$$\boldsymbol{\alpha_V = \frac{1}{T}}$$

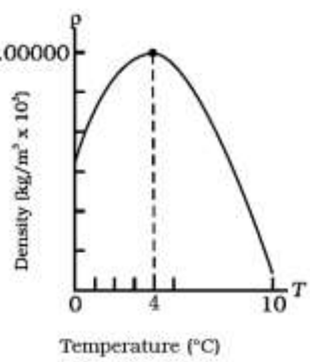
It shows that α_V decreases with increase in temperature.

What is anomalous expansion of water?

Water contracts on heating from 0^0C to 4^0C. This is known as anomalous expansion of water.

Draw the graph showing the variation of volume and density of water with temperature and explain in brief.





In the temperature range of 0^0C to 4^0C, volume of water decreases as temperature increases. Hence, Co-efficient of cubical expansion of water is negative. Hence water has greatest density at 4^0C.

Arrive at the relation between co-efficient of area expansion and co-efficient of linear expansion.

We have, $\frac{\Delta l}{l_0} = \alpha_l \, \Delta T$

$\Delta l = l_0 \, \alpha_l \, \Delta T$

$l - l_0 = l_0 \, \alpha_l \, \Delta T$

$l = l_0 + l_0 \, \alpha_l \, \Delta T$

$l = l_0(1 + \alpha_l \, \Delta T)$

Squaring on both sides, $l^2 = {l_0}^2(1 + \alpha_l \, \Delta T)^2$

$A = A_0\left(1 + 2\alpha_l \, \Delta T + \alpha_l^2 \, \Delta T^2\right)$

Since α_l is very small, $\alpha_l^2 \, \Delta T^2$ can be neglected.

$A = A_0(1 + 2\alpha_l \, \Delta T)$

Comparing with the equation, $A = A_0(1 + \alpha_A \, \Delta T)$

We have, $\boldsymbol{\alpha_A = 2\alpha_l}$

Arrive at the relation between co-efficient of volume expansion and co-efficient of linear expansion.

We have, $l = l_0(1 + \alpha_l \, \Delta T)$

Cubing on both sides, $l^3 = {l_0}^3(1 + \alpha_l \, \Delta T)^3$

$V = V_0\left(1 + 3\alpha_l \, \Delta T + 3\alpha_l^2 \, \Delta T^2 + \alpha_l^3 \, \Delta T^3\right)$

Since α_l is very small, $\alpha_l^2 \, \Delta T^2$ and $\alpha_l^3 \, \Delta T^3$ can be neglected.

$V = V_0(1 + 3\alpha_l \, \Delta T)$

Comparing with the equation, $V = V_0(1 + \alpha_V \, \Delta T)$

We have, $\boldsymbol{\alpha_V = 3\alpha_l}$

Further, $\boldsymbol{\alpha_l : \alpha_A : \alpha_V = \alpha_l : 2\alpha_l : 3\alpha_l = 1 : 2 : 3}$

Define heat Capacity (S). Mention its SI unit.

Heat capacity of a substance is defined as the amount of heat required to change the temperature of the substance by one unit.

$\boldsymbol{S = \frac{\Delta Q}{\Delta T}}$ where S is called Heat capacity

The S I unit is $J - K^{-1}$

Mention the factors on which the heat capacity of a substance depends.

The Quantity of heat required to warm a given substance also depends upon (i) its mass (m), (ii) change in temperature and (iii) nature of the material of the substance

Define Specific heat capacity (s). Mention its SI unit

Specific heat capacity of a substance is defined as the amount of heat required to change the temperature of unit mass of substance by one unit.

Specific heat capacity is also defined as heat capacity per unit mass.

$\boldsymbol{s = \frac{\Delta Q}{\Delta T} \frac{1}{m}}$ where s is called Specific heat capacity

The S I unit is $J \, kg^{-1} \, K^{-1}$

Mention the factors on which the specific heat capacity of a substance depends.

Specific heat capacity of a substance depends on, (i) nature of the material and (ii) raise or fall in temperature (ΔT)

Define Molar specific heat capacity. Mention its SI unit.

It is defined as the amount of heat required to change the temperature of one mole of substance by one unit.

$$\boldsymbol{C = \frac{\Delta Q}{\Delta T}\frac{1}{\mu}}$$

The S I unit is $J\,mol^{-1}\,K^{-1}$

Mention the additional conditions may be needed to define Molar specific heat capacity of substance.

When heat is supplied to a gas, the increase in temperature of the gas is accompanied either by increase in pressure or volume or both. Thus a gas can be heated under two conditions:
(i) at constant volume and (ii) at constant pressure

Why we may need additional conditions to define Molar specific heat capacity of substance?

Gases have very large co-efficient of expansion. Therefore, amount of heat supplied to a gas is used in two parts, (i) to raise the temperature of gas and (ii) to do mechanical work by the gas

Define Specific heat capacity at constant volume (C_V).

It is defined as, the amount of heat required to change the temperature of unit mass of gas by one unit at constant volume.

Define Specific heat capacity at constant pressure (C_P).

It is defined as, the amount of heat required to change the temperature of unit mass of gas by one unit at constant pressure.

Keep in mind

- Specific heat of water $s_w = 4186\,Jkg^{-1}K^{-1}$
- C_P is greater than C_V and $C_P - C_V = R$ This relation is called Mayer's relation.

What is change of state?

Matter normally exists in three states: solid, liquid and gas. A transition from one state to another is called change of state.

What is melting?

The change of state from solid to liquid is called melting.

What is melting point?

The temperature at which the solid and liquid co-exist in thermal equilibrium during change of state from solid to liquid is called melting point.

What is normal melting point?

The melting point of a substance at standard atmospheric pressure is called its normal melting point.

What is fusion?

The change of state from liquid to solid is called fusion or freezing.

What is effect of pressure on melting point?

The melting point decreases with increase in pressure.

What is Regelation? Give example.

It is the phenomenon in which ice melts when pressure is increased and again freezes when pressure is decreased.

Ex: When a metallic wire carrying two masses on either ends is hung from an ice cube, the wire passes through the ice cube without splitting it.

Explain how skating on ice is possible?

Skating is possible on snow due to the formation of water under the skates. Water is formed due to the increase of pressure and it acts as a lubricant.

What is vaporisation?

The change of state from liquid to vapour (gas) is called vaporisation.

What is boiling point?

The temperature at which the liquid and the vapour states of the substance co-exist is called boiling point.

What is normal boiling point?

The boiling point of a substance at standard atmospheric pressure is called its normal boiling point.

What is the effect of pressure on boiling point? Where this effect is used?

Boiling point of the substance increases with increase in pressure. This effect is used in the construction of pressure cooker.

Why cooking is difficult on hills?

At high altitudes, atmospheric pressure is lower and reduces the boiling point of water. This is why cooking is difficult on hills.

What is sublimation? Give example.

The change from solid state to vapour state without passing through the liquid state is called sublimation. Ex: Dry ice (solid Co_2), camphor etc.

What is Triple point?

The temperature at which the solid, liquid and vapour co-exist in thermal equilibrium is called triple point.

What is Latent heat? Mention the expression for it.

The amount of heat transferred per unit mass during the change of state of the substance is called latent heat.

If m is the mass of the substance that undergoes a change from one state to the other, then the quantity of heat required is given by,

$\boldsymbol{Q = mL}$ where $L \rightarrow$ latent heat

Mention the S I unit of Latent heat.

S I unit of latent heat is $J\ kg^{-1} (joule\ per\ kg)$

What is Latent heat of fusion (L_f)?

The amount of heat required to melt unit mass of solid completely at its melting point is called latent heat of fusion.

What is Latent heat of vaporisation (L_v)?

The amount of heat required to vaporise unit mass of liquid completely at its boiling point is called latent heat of vaporisation.

Draw the graph of temperature versus heat for a quantity of water OR
Plot a graph of temperature versus heat showing the changes in the states of ice on heating at one atmospheric pressure.

The graph between temperature and amount of heat supplied for water is as shown.

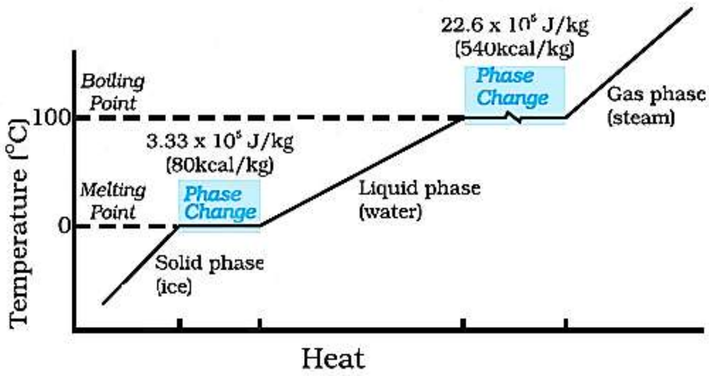


Illustration:

Steam burns are more serious than boiling water burns. Why?

The latent heat of vaporisation is $22.6 \times 10^5\ J\ kg^{-1}$. This heat gets released when steam changes its state from vapour to liquid. Hence steam burns are more serious.

What is Calorimetry?

The branch of science which deals with the measurement of heat is called Calorimetry.

What is the Principle Calorimetry?

Heat lost by the hot body is equal to the heat gained by the colder body, when they are kept in contact with each other, provided no heat is allowed to escape to the surroundings.

What is Calorimeter? Explain in brief.

A device in which heat measurement can be made is called Calorimeter.

Calorimeter is a hollow cylinder made of copper with a lid and a stirrer placed in it. The calorimeter is placed in an insulated enclosure so that there is no loss of heat by radiation.

Transfer of Heat

Mention the different modes of transfer of heat.

There are three modes of heat transfer, (i) Conduction, (ii) Convection and (iii) Radiation

What is conduction?

It is the mechanism of transfer of heat between two adjacent parts of a body because of their temperature difference.

How heat is transferred in conduction mode?

In this mode of heat transfer, transfer of heat takes place without any actual movement of the particles of the medium.

State and explain law of Thermal conductivity

The amount of heat that flows from hotter part to colder part is,

* directly proportional to area of cross-section (A) of the body.
* directly proportional to the temperature difference ($\Delta T = T_1 - T_2$) between the hotter part and colder part.
* directly proportional to the time (t) for which heat flows.
* inversely proportional to the distance (x) between the parts.

Thus we can write, $$Q \propto \frac{A\,(T_1 - T_2)\,t}{x}$$

$$\frac{Q}{t} = K\,\frac{A\,(T_1 - T_2)}{x}$$

$$\boldsymbol{H = K\,\frac{A\,(T_1 - T_2)}{x}}$$

where $K \rightarrow$ constant, called Thermal conductivity

Define Thermal conductivity. Mention its S I unit and dimensions.

It is defined as the rate of flow of heat per unit area of its surface normal to the direction of heat flow under unit temperature gradient.

S I unit of thermal conductivity is $J\,s^{-1}m^{-1}K^{-1}$ or $W\,m^{-1}K^{-1}$. Dimensions are $MLT^{-3}K^{-1}$

Mention some applications of thermal conduction.

* In winter, the iron chairs appear to be colder than the wooden chairs.
* Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
* We feel warm in woollen cloths.
* Houses made of concrete roofs get very hot during summer days.
* Steel utensils with copper bottom are good for uniform hearting of food.

Classify the substances based on thermal conductivity with examples.

Good conductors of heat	Bad conductors of heat
These substances have large values of thermal conductivity.	These substances have small values of thermal conductivity.
Most metals.	wood, air, wool, etc.
For Ideal conductor thermal conductivity, $K = \infty$	For ideal bad conductor thermal conductivity, $K = 0$

What is Convection?

It is the process in which heat is transferred by the actual movement of the particles of the medium.

Explain the formation of Winds.

In day time, earth is heated by the sun and hence air in contact with the earth gets heated up. This heated air, being lighter, rises up and is replaced by the cold and heavier air from large reservoir of water creating a sea breeze. At night this cycle is reversed forming land breeze.

Explain the formation of Trade winds.

The equatorial and polar regions of the earth receive unequal solar heat. Air at the earth surface near the equator is hot while the air in the upper atmosphere of the poles is cool. The cold air from the poles rushes towards the equator whose pressure is low. Thus, convection current of air starts between the equator and the poles. Due to rotation of earth from west to east, the convection current drifts towards the east. Convection current blows from North-east towards the equator, which is called Trade wind.

What is Radiation?

Radiation is the process in which heat is transferred from one region to another without the necessity of any intervening medium.

What is Radiant energy?

Energy emitter or radiated by a body during radiation in the form of electromagnetic waves is called Radiant energy.

What are thermal radiations?

Everybody emits energy in the form of waves due to its temperature. These waves are known as thermal radiations.

Mention the properties of thermal radiations.

* They travel along straight line at the seed of light.
* They can travel in vacuum.
* They do not heat intervening medium.
* They can be reflected and refracted.
* They exhibit the phenomenon of interference, diffraction and polarisation.
* They obey inverse square law that is their intensity varies inversely as the square of the distance from the source.

Keep in mind

* Conduction is possible both in solids and fluids.
* Convection is possible only in fluids.
* Convection can be natural or forced. Natural convection is responsible for many familiar phenomena such as sea breeze, land breeze, trade wind.
* Conduction and convection are not possible without any medium.
* Radiation does not requires any medium.
* Thermal radiations can be detected by thermopile, radiometer and bolometer.

Black body Radiation

What is a black body?

A body that absorbs all the radiations falling on it is called a black body.

What is black body radiation? Explain.

Radiations emitted by a black body are called black body radiations.

A black body at a given temperature emits all possible wavelengths at that temperature. The intensity and wavelength emitted are independent of the material of the black body but depend only on the temperature of the body.

(i) At low temperature, the wavelengths of the radiation emitted are in the infrared region. As the temperature of the black body is increased to about $1100K$, the emitted wavelength corresponds to the red region.

(ii) At sufficiently high temperatures ($3000K$), the emitted radiations contain shorter wavelengths.

(iii) The black body radiation consists of a continuous distribution of wavelengths covering infrared, visible and ultraviolet portions of the electromagnetic waves.

State and explain Wien's displacement law.

According to this law, the wavelength (λ_m), corresponding to maximum intensity of emission of black body radiation is inversely proportional to absolute temperature of the black body.

$$\lambda_m \propto \frac{1}{T}$$

$$\boldsymbol{\lambda_m T = b}$$

Where b is constant, called Wien's constant and $b = 2.898 \times 10^{-3} m\ K$

Mention the applications of Wien's displacement law.

(i) The colour of a piece of iron heated first becomes dull red, then reddish yellow and finally white hot. This can be explained using Wien's displacement law.

(ii) Wien's displacement law is used to find the temperature of Sun and stars.

State and explain Stefan's Law (Stefan-Boltzmann law).

Energy emitted by a black body per unit time per unit area is directly proportional to the fourth power of the temperature.

Mathematically, $\boldsymbol{H = A\,\sigma\,T^4}$

Where σ is called Stefan-Boltzmann constant and $\sigma = 5067 \times 10^{-8}\ Wm^{-2}K^{-4}$

What is Emissivity?

Emissivity of a body is defined as the ratio of the heat energy radiated per second per unit area by the body to the amount of heat energy radiated per second per unit area by a perfect black body at the same temperature.

Keep in mind

* Stefan's Law is obtained experimentally by Stefan and later proved theoretically by Boltzmann. Therefore it is also called as Stefan-Boltzmann law.
* For a body other than black body, the energy radiated per unit time is given by, $H = e\,A\,\sigma\,T^4$
* A body at temperature T, with surroundings at temperature T_S, emits as well as receives energy, the net rate of loss of radiant energy is, $H = A\,\sigma(\,T^4 - T_S^4)$
  Where $e \rightarrow$ emissivity of black body
* Emissivity of black body is one.

State and explain Newton's law of cooling.

The rate of loss of heat by a body is directly proportional to the temperature difference between the body and the surrounding.

$$-\frac{dQ}{dt} = k(T_2 - T_1)$$

where k is a positive constant depending upon the area and nature of the surface of the body.

T_2 is the temperature of the body and T_1 is the temperature of surroundings.

Show that $T_2 - T_1 = e^{-K\,t} + c'$ OR $T_2 = T_1 + e^{-K\,t}$ using Newton's law of cooling.

Consider a body of mass m and specific heat capacity s at temperature T_2.

Let T_1 be the temperature of the surroundings of the body.

According to Newton's law, $-\frac{dQ}{dt} \propto (T_2 - T_1)$

$$-\frac{dQ}{dt} = k(T_2 - T_1)$$

$$\frac{dQ}{dt} = -k(T_2 - T_1) \qquad --- (1)$$

Let the temperature of the body decreases by dT_2 in time dt.

Heat lost by the body is, $dQ = m\,s\,dT_2$

$$\frac{dQ}{dt} = m\,s\,\frac{dT_2}{dt} \qquad --- (2)$$

From equation (1)and (2), $\quad m\,s\,\frac{dT_2}{dt} = -k(T_2 - T_1)$

$$\frac{dT_2}{(T_2 - T_1)} = -\frac{k}{m\,s}dt$$

$$\frac{dT_2}{(T_2 - T_1)} = -K\,dt \qquad where\ K = \frac{k}{m\,s}$$

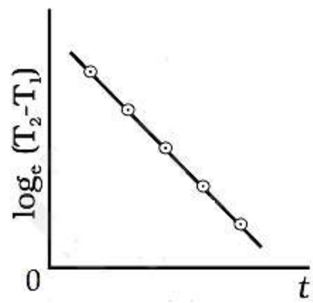
Integrating, $\quad \int \frac{dT_2}{(T_2 - T_1)} = -K\int dt$

$log_e(T_2 - T_1) = -K\,t + constant$

$\boldsymbol{T_2 - T_1 = e^{-K\,t} + c'}$ $\qquad$ where $c' = e^{constant}$

This equation shows a straight line having a negative slope.

The graph between $log_e(T_2 - T_1)$ and time t is as shown.



Chapter 11 THERMODYNAMICS

Thermal Equilibrium

What is Thermodynamics?

It is the branch of physics that deals with the concepts of heat and temperature and the inter conversion of heat and other forms of energy.

Mention the Macroscopic variables to specify the thermodynamics.

In thermodynamics, the main focus is on the macroscopic quantities of the system such as pressure, volume, temperature, mass, composition, internal energy, entropy, enthalpy etc. Thus, thermodynamics provide macroscopic description of the system.

How does thermodynamics differ from Mechanics?

Mechanics deals with motion of particles under the action of forces, while thermodynamics concerned with internal Microscopic state of the body.

What is thermodynamic system? Explain its types.

A collection of an extremely large number of atoms or molecules confined within certain boundaries such that it has certain values of pressure, volume and temperature is called thermodynamic system.

The system may be in the form of a solid, liquid, gas or a combination of two or more.

A thermodynamic system may be divided into three groups (types);

- **Open system:** It can exchange both energy and matter with its surroundings.
- **Closed system:** It can exchange only energy with its surroundings.
- **Isolated system:** It will not exchange both energy and matter with its surroundings.

What is meant by Surroundings in thermodynamics?

Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings.

What is an adiabatic wall?

A wall which does not allow any exchange of energy between the systems is known as adiabatic wall.

What is a diathermic wall?

A wall which allows any exchange of energy between the systems is known as adiabatic wall.

What are thermodynamic state variables (State variables)? Give examples.

Variables which are required to specify the state of thermodynamic system are called thermodynamic state variables.

Ex: Pressure, Temperature, Volume, Mass, Composition, Internal energy etc.

What is equation of state? Give the equation of state for an ideal gas.

The equation which relates the thermodynamic variables (state variables) is called equation of state.

The equation of state for an ***ideal gas*** is, $PV = \mu RT$

Mention the types of thermodynamic state variables.

State variables are of two types,

(i) Extensive thermodynamic state variable, (ii) Intensive thermodynamic state variable

What are extensive thermodynamic state variables? Give examples.

The variables whose value changes for each part of the system are called Extensive thermodynamic state variables.

Ex: Internal energy, volume and mass.

What are intensive thermodynamic state variables? Give examples.

The variables whose value remains unchanged for each part of the system are called intensive thermodynamic state variables.

Ex: Temperature, pressure and density.

What is thermal equilibrium?

Two systems in contact are said to be in thermal equilibrium, if both are at the same temperature.

In thermal equilibrium, thermodynamic variables such as pressure, volume, temperature, mass and composition will not change with time, for a closed system. That is the system has mechanical, thermal and chemical equilibrium.

Zeroth law of thermodynamics

State and explain Zeroth law of thermodynamics.

When two systems A and B are separately in thermal equilibrium with a third system C, then the two systems A and B are also in thermal equilibrium with each other.

Zeroth law was formulated by R H Flower.

Explanation: If A and B are two systems, equilibrium with system C then $T_A = T_C$ and $T_B = T_C$

This implies that $T_A = T_B$ i.e. the system A and B are also in thermal equilibrium.

What is the significance of Zeroth law of thermodynamics?

Significance of this law is, all the systems in thermal equilibrium with one another must have a common physical quantity that has the same value for both, called temperature.

Know the terms and concepts

Heat: Energy that is transferred between a system and its surroundings whenever there is a temperature difference between the system and surroundings is called heat.

When energy is transferred to the system from its surroundings, then heat is taken as positive. When energy is transferred to the surroundings from the system, then heat is taken as negative.

What is meant by work done by the system?

Work is said to be done, if a system moves through a certain distance in the direction of the applied force.

Obtain the expression for work done by the system.

Let a gas taken in the cylinder.

Let the cylinder is fitted with a frictionless piston of area of cross-section A.

Let P be the pressure of the gas on the cylinder.

The force on the piston, $F = PA$

Let the piston be displaced through a distance dx during the expansion of the gas.

Work done by the gas, $dW = Fdx$

$$dW = PAdx$$

$$dW = PdV$$

Total work done in which the volume changes from V_i to V_f is, $W = \int dW = \int_{V_i}^{V_f} PdV$

$$\boldsymbol{W = P(V_f - V_i)}$$

Keep in mind

Sign convention:

(i) When a system expands against the external pressure, $dV = (V_f - V_i)$ is positive. Hence work done by the system and is taken as positive.

(ii) When a system is compressed, dV is negative. Hence work done on the system and is taken as negative.

Define Internal energy.

The sum of kinetic and potential energy of the constituent particles of the system is known as internal energy. It is denoted by U.

$$U = U_K + U_P$$

Write the sign convention used in case of internal energy.

(i) Increase in internal energy is taken as positive and

(ii) Decrease in internal energy is taken as negative.

Mention the modes of changing internal energy.

Heat and work are modes of energy transfer to a system resulting in change in its internal energy.

First law of thermodynamics and its Applications

State and explain First law of thermodynamics.

When some quantity of heat (dQ) is supplied to a system, then the quantity of heat absorbed by the system is equal to the sum of the increases in the internal energy of the system (dU) and the external work done by the system (dW) against the expansion.

Mathematically, $dQ = dU + dW$

or $\boldsymbol{dQ = dU + PdV}$

Write the significance of first law of thermodynamics.

First law of thermodynamics is law of conservation of energy.

Using first law of thermodynamics, arrive at the Mayer's relation; $\boldsymbol{C_P - C_V = R}$.

From first law of thermodynamics, $dQ = dU + PdV$

For one mole of gas, If dQ is the heat absorbed at constant volume, then $dV = 0$

That is, $dQ = C_V\, dT$

or $C_V = \dfrac{dQ}{dT}$

$$C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{dU}{dT}\right)_V$$

or $C_V = \dfrac{dU}{dT} \qquad --- \ (1)$

Where the subscript V is dropped in the last step, since U of an ideal gas depends only on temperature.

If dQ is the heat absorbed at constant pressure, then

$$dQ = C_P\, dT$$

or $C_P = \dfrac{dQ}{dT}$

$$C_P = \left(\frac{dQ}{dT}\right)_P = \left(\frac{dU}{dT}\right)_P + \left(\frac{PdV}{dT}\right)_P$$

or $C_P = \dfrac{dU}{dT} + \left(\dfrac{PdV}{dT}\right)_P \qquad --- \ (2)$

The subscript P is dropped in the first tem, since U of an ideal gas depends only on temperature.

Now, for a mole of an ideal gas, $PV = RT$

or $PdV = RdT$

$$\left(P\frac{dV}{dT}\right)_P = R \quad --- (3)$$

Using the equations (1) and (3) in (2),

We have, $C_P = C_V + R$

$$\boldsymbol{C_P - C_V = R}$$

What is thermodynamic process?

Any process in which the thermodynamic variables of a system change is known as thermodynamic process.

What is Quasi-static process?

A process in which the system departs only infinitesimally from the equilibrium state is known as quasi-static process. In this process, the change in pressure or change in volume or change in temperature of the system is very, very small.

Keep in mind

Non-equilibrium states of a system are difficult to deal with. It is, therefore, convenient to imagine an ideal process in which at every stage the system is an equilibrium state. Such a process is infinitesimally slow, hence the name quasi-static.

What is an Isothermal process? Give examples.

A process in which the temperature of the system is kept constant throughout is called an isothermal process.

Ex: Boiling of a liquid, melting of wax or ice etc.

Explain isothermal process by applying first law of thermodynamics.

In this case P and V change, but $T = constant$.

As the temperature is constant, no change in internal energy, $dU = 0$.

From first law of thermodynamics, $dQ = dU + dW$

$$dQ = dW$$

or $$dQ = PdV$$

Heat supplied in an isothermal process is used to do work against the surrounding.

Obtain the expression for Work done during an Isothermal process.

Consider an ideal gas which changes its state from P_i, V_i to P_f, V_f at constant temperature.

The work done is given by, $W = \int dW = \int_{V_i}^{V_f} PdV$

$$W = \int_{V_i}^{V_f} \frac{\mu RT}{V}\, dV \qquad \text{(since } PV = \mu RT\text{)}$$

$$W = \mu RT \int_{V_i}^{V_f} \frac{1}{V}\, dV$$

$$W = \mu RT [\ln V]_{V_i}^{V_f}$$

$$\boldsymbol{W = \mu RT \ln\left[\frac{V_f}{V_i}\right]}$$

(i) If $V_f > V_i$ then $W = positive$. (ii) If $V_i > V_f$ then $W = negative$.

Know the term

Isotherm: The pressure-volume curve for a fixed temperature is called an isotherm.

What is adiabatic process? Give examples.

The process in which heat energy neither enters nor leaves the system is called adiabatic process.

Ex: Bursting of an automobile tube inflated with air, propagation of sound waves in a gas.

Explain adiabatic process by applying first law of thermodynamics.

In this case, P, V and T change, but $dQ = 0$.

From first law of thermodynamics, $dQ = dU + dW$

$$dW = -dU$$

When gas expands adiabatically, W is positive. Therefore dU must be negative. That is internal energy of the system decreases.

Obtain the expression for work done during an adiabatic process.

Let a gas in state P_i, V_i, T_i be adiabatically expand to the state P_f, V_f, T_f

Work done in the process is, $W = \int_{V_i}^{V_f} PdV$

For an adiabatic process, $PV^{\gamma} = constant$

$$W = \int_{V_i}^{V_f} \frac{constant}{V^{\gamma}} dV$$

$$W = constant \left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_i}^{V_f}$$

$$W = \frac{constant}{1-\gamma}\left[V_f^{1-\gamma} - V_i^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[constant \times V_f^{1-\gamma} - constant \times V_i^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[P_f V_f^{\gamma} V_f^{1-\gamma} - P_i V_i^{\gamma} V_i^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[P_f V_f - P_i V_i\right]$$

$$W = \frac{1}{1-\gamma}\left[\mu R T_f - \mu R T_i\right]$$

$$W = \frac{1}{1-\gamma}\mu R\left(T_f - T_i\right)$$

$$\boldsymbol{W = \frac{\mu R\left(T_i - T_f\right)}{\gamma - 1}}$$

(i) If $T_f < T_i$, $W > 0$ $(positive)$, Temperature decreases when the gas expands.

(ii) If $T_f > T_i$, $W < 0$ $(negative)$, Temperature increases when the gas compressed.

What is isochoric process? Give example.

A thermodynamic process that takes place at constant volume is called isochoric process.

Ex: Melting of a solid into liquid.

Explain isochoric process by applying first law of thermodynamics.

As the volume is kept constant, $dW = 0$

From first law of thermodynamics, $dQ = dU + dW$

$$dQ = dU$$

If heat is absorbed by a system at constant volume, its internal energy increases.

What is isobaric process? Give examples.

A thermodynamic process that takes place at constant pressure is called isobaric process.

Ex: Heating any liquid at atmospheric pressure, heating a gas at constant pressure.

Explain isobaric process by applying first law of thermodynamics.

For isobaric process pressure remains constant.

Work done by the gas is, $W = P\left(V_f - V_i\right)$

$$W = \mu R\left(T_f - T_i\right)$$

What is cyclic process?

It is the process in which the system returns to its initial state after a number of changes.

Explain cyclic process by applying first law of thermodynamics.

In cyclic process, change in internal energy is zero.

From the first law of thermodynamics, $dQ = dU + dW$

$$dQ = dW$$

Net work done during a cyclic process must be equal to the amount of heat energy transferred.

What is Reversible process? Give example.

It is a process which can be made to proceed in the opposite direction with same ease so that the system and the surroundings pass through exactly the same intermediate state as in the direct process.

Ex: Conversion of ice to water and vice versa, under ideal conditions.

What is irreversible process? Give example.

A process in which the system cannot be retraced to its original state is called an irreversible process.

Ex: A body moving on a rough surface from one point to another.

Second law of thermodynamics and Carnot engine

State both forms of second law of thermodynamics.

Kelvin-Planck statement: No process is possible whose only result is the absorption of heat from a reservoir and the complete conversion of heat into work.

Clausius statement: No process is possible whose only result is the transfer of heat from a colder object into a hotter object.

According to II law of thermodynamic, what are the limitations of efficiency and co-efficient of performance.

This law specifies the condition of transformation of heat into work. According the II Law,
Efficiency never be unity or never exceed unity.
Coefficient of performance never be infinite.

What is Carnot engine? Who designed the Carnot engine?

A reversible ideal heat engine operating between two temperatures is called a Carnot engine. Sadi Carnot introduced the concept of an ideal Carnot engine.

Explain the construction of Carnot engine.

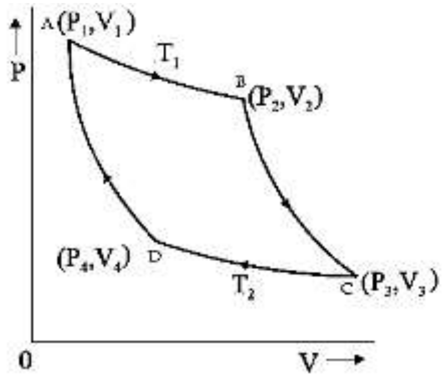
Parts of the Carnot engine are,

- Source: It is maintained at a fixed higher temperature T_1
- Sink: It is maintained at fixed low temperature T_2 than the source.
- Working substance: A perfect gas acts as working substance. The container is fitted with a piston which can slide without friction and it is also non-conducting. Container has conducting base and non-conducting side wall.
- Insulated stand: It is used to provide complete thermal isolation for working substance that can undergo adiabatic operation.

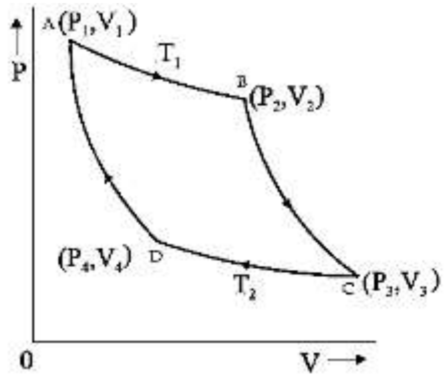
What is Carnot cycle? Write the schematic diagram representing Carnot cycle for a heat engine.

The working substance of the Carnot engine is taken through a cycle of isothermal and adiabatic process known as Carnot cycle.

Carnot cycle for a heat engine with an ***ideal gas as the working substance*** is as shown.



Explain different stages of Carnot's cycle with P-V diagram.



Step1- Isothermal expansion: The cylinder with gas having pressure P_1, volume V_1 and temperature T_1 is kept on the source at temperature T_1.

The gas is allowed to expand isothermally slowly.

The temperature tends to decrease, but it is maintained at constant temperature T_1 by absorbing heat from source.

Let the pressure and volume change to P_2 and V_2 respectively.

For an isothermal process, $dQ = dW$

$$Q_1 = W_1 = \mu R T_1 \ln\left(\frac{V_2}{V_1}\right)$$

Step2-Adiabatic expansion: The cylinder is placed on the non-conducting stand and the gas is allowed to expand adiabatically until the temperature falls to T_2 .

Work done during the expansion is, $W_2 = \dfrac{\mu R}{\gamma - 1}\ (T_1 - T_2)$

To make the gas recover its capacity for doing work, it should be brought to the original condition. This is done in next steps.

Step3-Isothermal compression:

The cylinder is kept on sink at temperature T_2.

The gas is compressed isothermally.

Let Q_2 amount of heat is rejected to the sink.

Let the pressure P_3 and volume V_3 change to P_4 and V_4 respectively.

For an isothermal process, $dQ = dW$

$$Q_2 = W_3 = -\mu R T_2 \ln\left(\frac{V_4}{V_3}\right)$$

$-ve$ sign indicates work is done on the system.

$$Q_2 = W_3 = \mu R T_2 \ln\left(\frac{V_3}{V_4}\right)$$

Step4-Adiabatic Compression: The cylinder is placed on the non-conducting stand and the gas is compressed adiabatically till the pressure P_4, volume V_4 changes to P_1 and V_1 and temperature T_1.

Work done, $W_4 = \dfrac{\mu R}{\gamma - 1}\ (T_1 - T_2)$

Total work done during the complete cycle is, $W = W_1 + W_2 - W_3 - W_4$

Since $W_2 = W_4$, $W = W_1 - W_3 = Q_1 - Q_2$

Obtain the expression for Efficiency of Carnot engine.

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{\mu R T_2 \ln\left(\frac{V_3}{V_4}\right)}{\mu R T_1 \ln\left(\frac{V_2}{V_1}\right)}$$

$$\eta = 1 - \frac{T_2 \ln\left(\frac{V_3}{V_4}\right)}{T_1 \ln\left(\frac{V_2}{V_1}\right)} \qquad --- \ (1)$$

For the step2, we have adiabatic equation, $T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$

$$\frac{T_2}{T_1} = \frac{V_2^{\gamma-1}}{V_3^{\gamma-1}} \qquad --- \ (2)$$

For Step4, we have adiabatic equation, $T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$

$$\frac{T_2}{T_1} = \frac{V_1^{\gamma-1}}{V_4^{\gamma-1}} \qquad --- \ (3)$$

From equation (1) and (2), $\frac{V_2^{\gamma-1}}{V_3^{\gamma-1}} = \frac{V_1^{\gamma-1}}{V_4^{\gamma-1}}$

$$\left(\frac{V_2}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$$

$$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

or $$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\ln\left(\frac{V_2}{V_1}\right) = \ln\left(\frac{V_3}{V_4}\right)$$

Equation (1) becomes, $\boldsymbol{\eta = 1 - \frac{T_2}{T_1}}$

State Carnot's theorem <u>OR</u> Mention the important conclusions established from the expression for efficiency of Carnot engine.

(a) No engine can have efficiency more than that of the Carnot engine working between two given temperatures T_1 and T_2 of the hot and cold reservoirs respectively and

(b) The efficiency of the Carnot engine is independent of the nature of the working substance.

Keep in mind

- Efficiency of Carnot engine depends upon the temperature of the source and sink.
- Efficiency is independent of the nature of the working substance.
- Since we cannot have a sink at absolute zero, so a heat engine with 100% efficiency is not possible to realise in practice.

Chapter-12 **KINETIC THEORY**

Molecular nature of matter and Behaviour of gas

Who developed Kinetic Theory?
Kinetic theory was developed by Maxwell, Boltzmann and Gibbs.

How Kinetic theories explain the behaviour of gas?
It explains the behaviour of gases, based on the idea that, gases consists of a large number of atoms or molecules, which are in the state of continuous random motion and the interatomic forces binding the atoms are negligible.

What are the reasons for the Success of Kinetic theory?
Kinetic theory was successful due to,
- It gives a molecular interpretation of pressure and temperature of a gas.
- It is consistent with gas laws and Avogadro's hypothesis.
- It is correctly explains specific heat capacities of many gases.
- It relates measurable properties of gases such as viscosity, conduction and diffusion with molecular parameters such as molecular size and mass.

How molecular nature of matter was established?
Molecular nature of matter can be established by considering the following concepts.
Atomic Hypothesis, Atomic Theory and Avogadro's Hypothesis.

State Atomic Hypothesis.
All the things are made of atoms – which are tiny particles that move around in perpetual motion, attracting each other when they are little distance apart but repelling upon being squeezed into one another.

Who suggested the Atomic hypothesis?
Kanada in India and Democritus' in Greece has suggested the Atomic hypothesis.

Who proposed Atomic Theory?
John Dalton proposed atomic theory.

Write the postulates of Dolton's atomic theory.
(i) The smallest constituents of an element are atoms.
(ii) Atoms of one element are identical but differ from those of other element.
(iii) A small number of atoms of each element combine to form a molecule of the compound.

State Gay Lussac's Law.
When gases combine chemically to yield another gas, their volumes are in the ratio of small integers.

Explain the behaviour of gas.
Gases at low densities, low pressures and high temperatures obey the experimental result,

$$\frac{PV}{T} = K$$

Where K is a constant and depends on mass of the gas, hence $K \propto N$

or $K = k_B N$ where $N \rightarrow$ Number of molecules, $k_B \rightarrow$ Boltzmann constant $= 1.38 \times 10^{-23}\ J\ K^{-1}$

Implies that, $\boldsymbol{\frac{PV}{T} = k_B N}$

State and explain Avogadro Hypothesis.

The number of molecule per unit volume is same for all gases at a fixed temperature and pressure.

Explanation: We have,

$$\frac{P_1V_1}{T_1N_1} = \frac{P_2V_2}{T_2N_2} = k_B$$

If P, V and T are same, then N is also same for all gases and this number is denoted as N_A, called Avogadro Number. In 22.4 litres of any gas, $N_A = 6.02 \times 10^{23}$.

Hence the equation becomes, $\boldsymbol{PV = k_B N_A T}$

What is meant by 1 mole of substance?

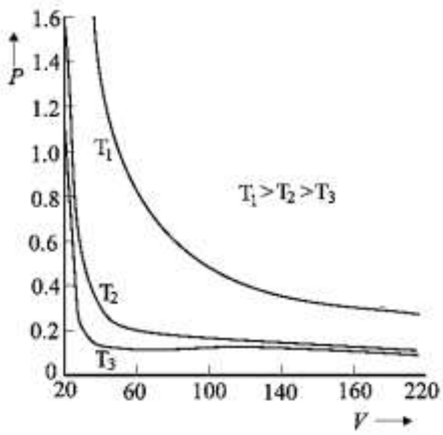
The mass of 22.4 litres any gas is equal to its molecular weight in grams at STP. This amount is called as 1 mole of substance.

State and explain Boyle's Law.

At constant temperature, pressure of a given mass of gas varies inversely with its volume.

If T is constant, $P \propto {}^1/_V$ or $PV =$ constant

Draw P-V curve or diagram for Boyle's law.

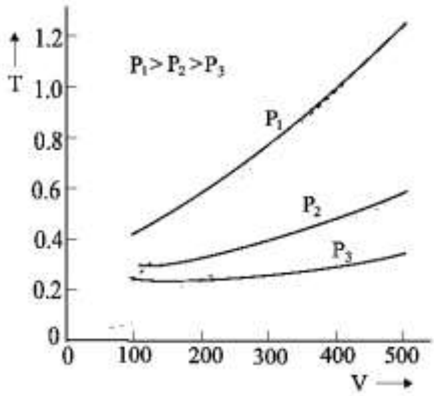


State and explain Charles' Law.

At constant pressure, the volume of the gas is proportional to its absolute temperature.

If P is constant, $V \propto T$ or ${}^V/_T$ = constant

Draw T-V curve or diagram for Charles' law



What is an Ideal Gas?

A gas in which the molecules do not exert any attractive or repulsive force on each other is called an Ideal gas or Perfect gas. ***or***

A gas which obeys Boyle's law and Charles' law is called Ideal gas or Perfect gas.

Arrive at the Ideal gas equation.

For one mole of gas, we have

$PV = \text{constant}$ (Boyle's law)

$V/T = \text{constant}$ (Charles' law)

Combining these, we get $\frac{PV}{T} = constant$

The constant is equal to R and known as Universal gas constant, its value is 8.31 $J\ mol^{-1}K^{-1}$.

So, $PV = RT$

For μ moles, $\boldsymbol{PV = \mu RT}$

Obtain the expression for universal gas constant and mention the value of it.

We have $PV = RT$

and $PV = k_B N_A T$

On comparison we get, $\boldsymbol{R = k_B N_A}$ $k_B \rightarrow$ Boltzmann constant and $N_A \rightarrow$ Avogadro number

The value of R is 8.31 $J\ mol^{-1}K^{-1}$

Deduce the expression $\boldsymbol{P = \frac{\rho RT}{M_0}}$

We have $\mu M_0 = M$

$M \rightarrow$ Mass of the gas and $M_0 \rightarrow$Molar mass

$$\mu = \frac{M}{M_0}$$

Substituting the value of μ in $PV = \mu RT$ we get,

$$PV = \frac{M}{M_0}RT$$

$$P = \frac{M}{V}\frac{RT}{M_0}$$

$$\boldsymbol{P = \frac{\rho RT}{M_0}}$$

Kinetic theory of an ideal gas

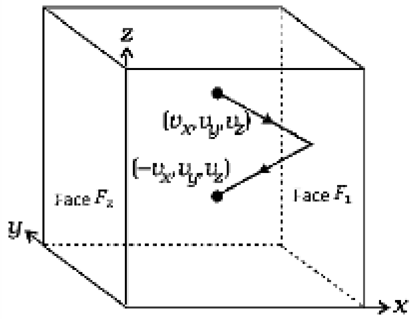
Mention the assumptions of Kinetic Theory.

1) Any gas consists of a very large number of identical particles called molecules, each having identical mass.
2) The molecules are considered to be rigid, perfectly elastic solid spheres of negligible size.
3) The molecules are in a state of random motion, moving in all directions with all possible velocities.
4) All the collisions between the molecules or between a molecule and the wall of the container are perfectly elastic.
5) The molecules exert no force of attraction or repulsion on each other and with the walls of the container, except during the collision.

6) The molecules during motion collide with one another and with the walls of the container. Between collisions, the molecules move in a straight line with uniform velocity. At each collision their velocity gets altered.
7) In steady state, the collisions do not affect the molecular density of the gas.
8) The molecules obey Newton's laws of motion.

Derive $P=\frac{1}{3}nm\overline{V^2}$ with usual notation <u>OR</u> Derive an expression for pressure of an ideal gas.

Consider a gas enclosed in a cube of side l.
Let n be the number of molecules per unit volume.
Let a molecule of mass m moving with velocity (v_x, v_y, v_z) along x −axis hits the face F_1.
Since the collision is elastic, the x − component of velocity of the molecule is reversed whereas the y and z components remain unchanged.



Initial momentum of the molecule along x −axis $= mv_x$
Final momentum of the molecule along x −axis $= -mv_x$
Change in momentum $= -mv_x - mv_x = -2mv_x$
Momentum transferred to the wall $= 2mv_x$

To calculate pressure on the face F_1, we have to calculate momentum transferred in time dt.
In time dt, a molecule with x −component of velocity v_x will hit the face F_1, if it is within a distance of $v_x\, dt$ from face F_1.
Then all the molecules within this distance will hit the face F_1.
Number of molecules within this volume $= n\,A\,v_x\,dt$
On the average half of these molecules are moving towards face F_1 and half away from the face F_1
Number of molecules hitting the face $F_1 = \frac{1}{2}\,n\,A\,v_x\,dt$
Momentum transferred to the wall by these molecules $dp = 2mv_x\left(\frac{1}{2}\,n\,A\,v_x\,dt\right)$

$$dp = n\,mA\,v_x^2\,dt$$

Force, $F = \frac{dp}{dt} = n\,mA\,v_x^2$

Pressure, $P = \frac{F}{A} = \frac{n\,mA\,v_x^2}{A} = n\,m\,v_x^2$

Actually all the molecules in the gas do not have the same velocity.
Therefore Pressure exerted by these molecules on face F_1 is, $P = n\,m\,\overline{v_x^2}$ where $\overline{v_x^2} \rightarrow$ average of v_x^2
But Inside the container the molecules are in random motion.
By symmetry $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

Average of $\frac{\overline{v_x^2}+\overline{v_y^2}+\overline{v_z^2}}{3} = \frac{1}{3}\overline{v^2}$

Pressure, $\boldsymbol{P = \frac{1}{3}\,n\,m\,\overline{v^2}}$ where $\overline{v^2} \rightarrow$ mean of the square speed.

Derive the relation between kinetic energy of a gas molecule and its absolute temperature.

OR

From kinetic theory of gases explain the kinetic interpretation of temperature.

We have $P = \frac{1}{3}\,n\,m\,\overline{v^2}$

$$PV = \frac{1}{3}\, nV\, m\, \overline{v^2}$$

$$PV = \frac{1}{3}\, N\, m\, \overline{v^2}$$

$$PV = \frac{2}{3}\, N\, \left(\frac{1}{2} m\, \overline{v^2}\right) \qquad \rightarrow \text{(i)}$$

Here $\frac{1}{2} m\, \overline{v^2}$ is the average translational kinetic energy of the molecules of the gas.

The internal energy of the gas is purely kinetic and is given by, $E = N\, \frac{1}{2}\, m\, \overline{v^2}$

Equation (i)becomes, $PV = \frac{2}{3}\, E$

But we have, $PV = k_B NT$

Therefore $E = \frac{3}{2}\, k_B NT$

$$\boldsymbol{\frac{E}{N} = \frac{3}{2} k_B T}$$

Where $\frac{E}{N} \rightarrow$ average kinetic energy of a molecule.

Implies that, $\boldsymbol{\frac{E}{N} \propto T}$ or $\boldsymbol{\langle E_t \rangle \propto T}$

The average kinetic energy is directly proportional to the absolute temperature.

Define temperature based on Kinetic theory.

Temperature is defined as the average kinetic energy of a molecule.

What is RMS speed of a gas molecule?

It is the square root of the mean of the square of the velocities of individual molecules of the gas.

Arrive at the expression for RMS speed.

We have $\frac{1}{2} m\, \overline{v^2} = \frac{3}{2} k_B T$

$$\overline{v^2} = 3\, \frac{k_B T}{m}$$

$$\boldsymbol{v_{rms} = \sqrt{\frac{3\, k_B T}{m}}} \qquad \text{Where} \quad v_{rms} = \sqrt{\overline{v^2}}$$

Degrees of freedom and law of equipartition energy

Define degrees of freedom of a molecule.

The number of co-ordinates required to specify the configuration and position of a gas molecule is called Degrees of freedom. **or**

It is the total number of independent ways in which the gas molecule can absorb the energy.

How many degrees of freedom a mono atomic gas molecule possess? Explain OR

By applying law of Equipartition of energy determine degrees of freedom a mono atomic gas molecule?

A monoatomic gas molecule consists of a single atom. It can have translational motion in any direction in 3-dimensional space. Therefore it has 3 translational degrees of freedom (n=3).

$$\therefore\ E_t = \frac{1}{2}\,m\,v_x^2 + \frac{1}{2}\,m\,v_y^2 + \frac{1}{2}\,m\,v_z^2$$

How many degrees of freedom a di-atomic gas molecule possess? Explain OR
By applying law of Equipartition of energy determine degrees of freedom di-atomic gas molecule?

The molecule consists of two atoms bound to each other. Assuming that the diatomic molecule is rigid, it has,

a) 3 translational degrees of freedom. (Each along x, y and z −axis)
b) 2 rotational degrees of freedom. (One along y and other z −axis)
c) No vibrational energy. (because molecule is rigid)

Therefore $n = 3 + 2 = 5$

$$\therefore\ E_t = \frac{1}{2}\,m\,v_x^2 + \frac{1}{2}\,m\,v_y^2 + \frac{1}{2}\,m\,v_z^2 + \frac{1}{2}\,I_y\omega_y + \frac{1}{2}\,I_z\omega_z$$

Mention the expression for number of degrees of freedom of a molecule and explain the terms.

$$\boldsymbol{n = 3N - r}$$

Number of degrees of freedom of a molecule is equal to total number of co-ordinates required to specify the positions of the molecule(N) minus the number of independent relations between the atoms or molecules(r).

How many degrees of freedom a non-linear rigid tri atomic gas molecule possess?

For a triatomic molecule (non-linear structure), $n = 3N - r$

$N = 3$ and $r = 2$ implies that, $n = 3(3) - 3 = 6$

A non-linear rigid triatomic molecule has 3 translational degrees of freedom, 3 rotational degrees of freedom and no vibrational degrees of freedom.

What is internal energy of an Ideal gas. Explain.

Internal energy of an ideal gas is sum of potential and kinetic energies of all the gas molecules. Denoted by U.

$$U = K + V$$

For an ideal gas, potential energy is zero, since no intermolecular forces between the molecules and kinetic energy may be translational, rotational and vibrational.

$V = 0\ and\ K = E_t + E_r + E_v$ then $U = K$

State and explain law of Equipartition of energy.

In thermal equilibrium, the total energy is equally distributed in all possible degrees of freedom and average energy in each degree of freedom is equal to $\frac{1}{2}k_BT$.

Explanation: The translational kinetic energy of a single molecule is,

$$E_t = \frac{1}{2}\,m\,v_x^2 + \frac{1}{2}\,m\,v_y^2 + \frac{1}{2}\,m\,v_z^2$$

The average energy of the gas is given by, $\langle E_t\rangle = \langle\frac{1}{2}\,m\,v_x^2\rangle + \langle\frac{1}{2}\,m\,v_y^2\rangle + \langle\frac{1}{2}\,m\,v_z^2\rangle$

From Kinetic theory, $\frac{E}{N} = \langle E\rangle = \frac{3}{2}k_BT$

$$\langle\frac{1}{2}\,m\,v_x^2\rangle + \langle\frac{1}{2}\,m\,v_y^2\rangle + \langle\frac{1}{2}\,m\,v_z^2\rangle = \frac{3}{2}k_BT$$

$$\langle\frac{1}{2}\,m\,v_x^2\rangle = \frac{1}{2}k_BT, \qquad \langle\frac{1}{2}\,m\,v_y^2\rangle = \frac{1}{2}k_BT, \qquad \langle\frac{1}{2}\,m\,v_z^2\rangle = k_BT$$

Illustration:

(i) Calculate the average kinetic energy of oxygen molecule at $0^0\ C$.

Oxygen is diatomic molecule and has 5 degrees of freedom.

$$\langle E\rangle = \frac{5}{2}k_BT = \frac{5}{2}\frac{R}{N_A}T = \frac{5}{2}\times\frac{8.314}{6.023\times 10^{23}}\times 273 = 9.4\times 10^{-21}\,J$$

(ii) Calculate the ratio of average kinetic energy of molecule of oxygen and neon gas at $27^0\ C$.

Oxygen is diatomic molecule and has 5 degrees of freedom.

$$\langle E\rangle_O = \frac{5}{2}k_BT$$

Neon is a monoatomic molecule and has 3 degrees of freedom.

$$\langle E\rangle_{Ne} = \frac{3}{2}k_BT$$

$$\frac{\langle E\rangle_O}{\langle E\rangle_{Ne}} = \frac{5k_BT}{2}\times\frac{2}{3k_BT} = \frac{5}{3}$$

Determine the specific heat capacity of a mono atomic gas molecule and hence find ratio of specific heats.

Molecule of monoatomic gas has 3 degrees of freedom.

Total internal energy of the monoatomic gas, $U = \frac{3}{2}k_BTN_A$

$$U = \frac{3}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{3}{2}R$$

$$C_P - C_V = R$$

$$C_P = R + C_V = R + \frac{3}{2}R = \frac{5}{2}R$$

The ratio of specific heats, $\boldsymbol{\gamma = \frac{C_P}{C_V} = \frac{5R}{2}\times\frac{2}{3R} = \frac{5}{3}}$

Determine the specific heat capacity of a di-atomic gas molecule assuming as a rigid rotator and hence find ratio of specific heats.

Molecules of diatomic gas have 5 degrees of freedom.

$$U = \frac{5}{2}k_BTN_A$$

$$U = \frac{5}{2}RT$$

$$C_V = \frac{dU}{dT} = \frac{5}{2}R$$

$$C_P - C_V = R$$

$$C_P = R + C_V = R + \frac{5}{2}R = \frac{7}{2}R$$

The ratio of specific heats, $\boldsymbol{\gamma = \frac{C_P}{C_V} = \frac{7R}{2}\times\frac{2}{5R} = \frac{7}{5}}$

Deduce the equation $\boldsymbol{\gamma = \frac{(4+f)}{(3+f)}}$ for polyatomic gases.

A poly molecule has 3 translational, 3 rotational and f number of vibrational energy.

$$U = \left(\frac{3}{2}k_BT + \frac{3}{2}k_BT + f\,k_BT\right)N_A$$

$$U = \left(\frac{3}{2} + \frac{3}{2} + f\right) N_A k_B T$$
$$U = (3 + f)RT$$
$$C_V = \frac{dU}{dT} = (3 + f)R$$
$$C_P = R + C_V$$
$$C_P = R + (3 + f)R = (4 + f)R$$
$$\boldsymbol{\gamma = \frac{C_P}{C_V} = \frac{(4 + f)}{(3 + f)}}$$

Show that the specific heat capacities of solids is 3R

Every atom in a solid vibrates about its equilibrium position.

A one dimensional oscillator has two degrees of freedom, one translational and one vibrational.

The average energy $= 2 \times \frac{1}{2} k_B T = k_B T$

Each atom in a solid can be treated as three dimensional harmonic oscillators.

Hence average energy of an atom in a solid = $3k_B T$

If there are N_A atoms, total energy per mole, $U = 3k_B T N_A = 3RT$

At constant pressure we have $dQ = dU + PdV$

But $dV \approx 0$ for solids, then $dQ = dU$

$$\boldsymbol{C = \frac{dU}{dT} = \frac{dQ}{dT} = 3R}$$

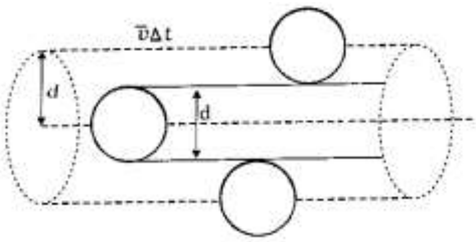
Mean free path

What is free path?

The distance travelled by a gas molecule between two successive collisions is known as free path.

What is mean free path of a gas molecule?

The average distance travelled by a molecule between two successive collisions is called mean free path.



Obtain the expression for mean free path.

Consider a gas containing n molecules per unit volume.

Let d be the diameter of the each molecule.

Consider one such molecule is in motion and other are at rest.

The moving molecule will collide with all those molecules whose centres are at a distance d from the centre of the molecule.

Let $\langle v \rangle$ be the average speed and distance travelled by the molecule in time Δt is $\langle v \rangle \Delta t$.

This molecule collide with all the molecules whose centres lie on the cylinder of radius d.

Number of molecules in this volume, $N = n\,\pi\,d^2\,\langle v \rangle\,\Delta t$

Number of collisions in this volume = Number of molecules = $n\,\pi\,d^2\,\langle v \rangle\,\Delta t$

Rate of collision $= \dfrac{Number\ of\ collisions}{\Delta t} = \dfrac{n\,\pi\,d^2\,\langle v \rangle\,\Delta t}{\Delta t} = n\,\pi\,d^2\,\langle v \rangle$

Average time between two successive collisions, $\tau = \dfrac{1}{Rate\ of\ collision}$

$$\tau = \frac{1}{n\,\pi\,d^2\,\langle v\rangle}$$

Average distance between two successive collisions – mean free path, $l = \langle v\rangle\,\tau$

But actually all molecules are moving.

So collision rate is determined by average relative velocity of the molecule.

Therefore, mean free path, $l = \langle v_r\rangle\,\tau$

But $\langle v_r\rangle = \frac{\langle v\rangle}{\sqrt{2}}$

Men free path, $l = \frac{\langle v\rangle}{\sqrt{2}}\,\tau = \frac{\langle v\rangle}{\sqrt{2}}\,\frac{1}{n\,\pi\,d^2\,\langle v\rangle}$

$$\boldsymbol{l = \frac{1}{\sqrt{2}\;n\,\pi\,d^2}}$$

Mention the factors on which mean free path of gas molecule depends?

Mean free path is inversely proportional to number of molecules per unit volume, (n) and size of the molecule (d).

Chapter-13 OSCILLATIONS

Periodic and Oscillatory Motion

What is Periodic motion? Give example.
A motion that repeats itself at regular intervals of time is called periodic motion.
Ex: Motion of planets in solar system, uniform circular motion.

What is Oscillatory motion? Give example.
A motion in which a body moves to and fro between two extreme positions about an equilibrium position is called oscillatory motion.
Ex: boat tossing up and down, piston of a steam engine, motion of simple pendulum.

What is the equilibrium (mean) position of an oscillation body?
It is the position of a body during oscillatory motion at which the net external force acting on the body is zero **OR** It is the position, at which if it is at rest, it remains at rest forever.

Keep in mind
Oscillations or vibrations: The motion of a body between two extreme positions forms oscillations or vibrations.
(i) There is no significant difference between oscillations and vibrations. When the frequency is small we call it oscillation, while the frequency is high we call it vibrations.
(ii) Every oscillatory motion is periodic; but every periodic motion need not be oscillatory.

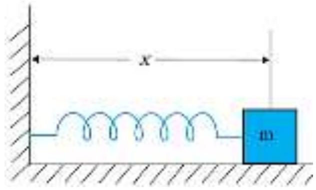
Importance of oscillatory motion: This motion is basic to physics. In musical instruments we come across vibrating strings, membranes in drums and diaphragms in telephone and speaker system vibrate, vibrations of air molecule, vibrations of atoms in solid include oscillatory motion. The concepts of oscillatory motion are required to understand many physical phenomena listed above.

Define Period or Time period (T). Mention its SI unit.
The smallest interval of time after which a periodic motion repeats is called period.
In case of oscillation, the time taken by the body to complete one oscillation is called period.
SI unit of period is $second$.

Define Frequency (ν). Mention its SI unit.
Number of times a periodic motion repeats per unit time is called frequency.
In case of oscillations, number of oscillations per unit time is called frequency.
SI unit of frequency is $hertz$ (Hz). $1Hz = 1$ oscillation per second.

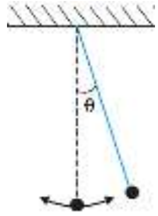
Give the relation between period and frequency of periodic motion.
Relation between period and frequency is given by, $T = {}^{1}/_{\nu}$ or $\nu = {}^{1}/_{T}$

What is Displacement (x or y) in periodic motion?
The term displacement refers to change of physical quantity with time.
In periodic motion displacement may be linear as well as angular.



What is linear displacement?
The straight line distance travelled by a particle from its equilibrium position.

What is angular displacement?
It is the angle through which position vector of the body rotates in a given time.



Define amplitude (A).
The maximum displacement of the particle from its equilibrium position is called amplitude.

Illustration:
(i) On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.

Period, $T = \frac{Total\ time}{No.of\ oscillations} = \frac{60\ s}{75} = 0.8\ s$

Frequency, $\nu = \frac{1}{T} = \frac{1}{0.8} = 1.25\ Hz$

(ii) A particle takes 32 s to make 20 oscillations. Calculate time period and frequency.

Period, $T = \frac{32}{20} = 1.6\ s$

Frequency, $\nu = \frac{1}{T} = \frac{1}{1.6} = 0.625\ Hz$

What is a periodic function? Give example.
Any function which repeats itself after a regular interval of time is called periodic function.
In periodic motion displacement is periodic function and it can be represented by a mathematical function of time. The simplest of these functions is given by, $f(t) = A\cos\omega t$.

Show that the value of the function remains same for a periodic function.
We have $f(t) = A\cos\omega t$ where ωt is called the angular displacement.
In one revolution, the angle covered (angular displacement) by the reference particle is 2π and time period is T.
If ωt is increased by an integral multiple of 2π radian, then

$$f(t+T) = A\cos(\omega t + 2\pi)$$
$$f(t+T) = A\cos\omega t = f(t)$$

Hence the value of periodic function remains same.

Keep in mind
(i) In $\cos\omega t$, the term ω is called angular frequency.
Angular frequency, $\omega = \frac{2\pi}{T}$ and SI unit is $radian\ per\ second$.
(ii) The function $f(t) = A\sin\omega t$ is also periodic.
(iii) The linear combination of both sine and cosine function is also periodic and it is represented by $f(t) = A\sin\omega t + B\cos\omega t$ and it is called Fourier series.
By putting, $A = D\cos\phi$ and $B = D\sin\phi$

$$f(t) = D \sin \omega t \cos \phi + D \cos \omega t \sin \phi$$

$$f(t) = D \sin(\omega t + \phi) \qquad \text{where } D = \sqrt{A^2 + B^2} \text{ and } \phi = tan^{-1}\left(\frac{B}{A}\right)$$

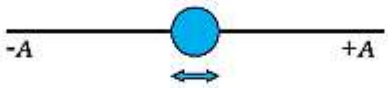
Simple Harmonic motion

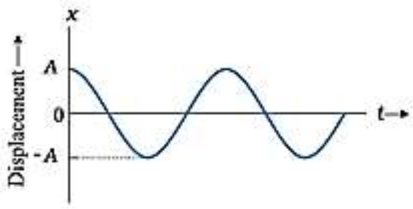
Define Simple harmonic motion (SHM).

The oscillatory motion is said to be simple harmonic, if the displacement of the particle from the origin varies with time as; $x(t) = A\ \cos(\omega t + \phi)$ or $y(t) = A \sin(\omega t + \phi)$.

Explain simple harmonic motion with and example and draw the displacement time graph.

Consider a particle oscillating back and forth about the origin along $x - axis$ between the limits $+A$ and $-A$ as shown. Here the motion of the particle is simple harmonic motion in which displacement is a sinusoidal function of time.





What is Phase in periodic motion?

During the periodic motion, the position and velocity of the particle at any time t is determined by the term $(\omega t + \phi)$ in cosine function. This quantity is called phase of the motion.

What is Phase constant (Phase angle)?

In term the term $(\omega t + \phi)$ the value of phase at $t = 0$ is ϕ and it is called the phase constant or phase angle.

Illustration:

Find the time taken by the particle in going from $x = 0$ to $x = {}^{A}/_{2}$ where A is amplitude.

$$x(t) = A\ \cos(\omega t + \phi)$$

$$\frac{A}{2} = A \cos(\omega t + 0)$$

$$\frac{A}{2} = A \cos\left(\frac{2\pi}{T}t\right) \Rightarrow \frac{1}{2} = \cos\left(\frac{2\pi}{T}t\right) \Rightarrow \cos\frac{\pi}{3} = \cos\left(\frac{2\pi}{T}t\right)$$

$$\frac{\pi}{3} = \frac{2\pi}{T}t \Rightarrow t = \frac{T}{6}$$

Show that the projection of uniform circular motion on a diameter of the circle follows simple harmonic motion <u>OR</u> Compare Simple harmonic motion and uniform circular motion.

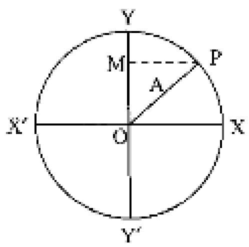
Consider a particle moving with a uniform sped along the circumference of circle of radius A.

Let the particle start from the point X with a constant speed ω.

After some time it reaches to P.

Draw PM perpendicular to $y - axis$.

OM represents the projection of position vector of the particle on $y - axis$.



When the particle moves from X to Y its projection of the position vector moves from O to Y. As the particle moves from Y to X', its projection moves from Y to O. Similarly the particle moves from X' to X via Y', its projection moves from O to Y' and Y' to O. This shows that if the particle moves uniformly on a circle, its projection on the diameter ($y - axis$) of the circle executes SHM.

The position of the particle on the circle is given by, $x(t) = A\cos(\omega t + \phi)$

The displacement of the projection on $y - axis$ is given by, $y(t) = A\sin(\omega t + \phi)$ which is also SHM with same amplitude but different in phase by $\pi/_2$.

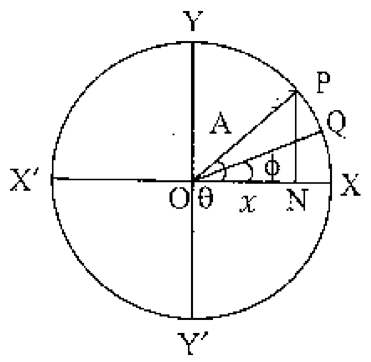
Arrive at the equation of SHM.

Consider a particle moving on a circle of radius A with uniform velocity ω.

Let the particle start from X and subtend an angle θ in time t and reaches P.

Angular velocity, $\omega = \frac{\theta}{t}$

$$\theta = \omega t$$



The projection of the particle on $x - axis$ is, $ON = x$

In ΔOPM, $\cos\theta = \frac{ON}{OP} = \frac{x}{A}$

$$x = A\cos\theta$$

$$\boldsymbol{x = A\cos\omega t}$$

If the particle starts from Q, $\boldsymbol{x = A\cos(\omega t + \phi)}$

Obtain the expression for velocity of the particle executing SHM.

We have, $x = A\cos(\omega t + \phi)$

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

Further, $v = -A\omega\sqrt{1 - \cos^2\omega t}$

$$v = -A\omega\sqrt{1 - \left(\frac{x}{A}\right)^2}$$

$$v = -A\omega\sqrt{\frac{A^2 - x^2}{A^2}}$$

$$\boldsymbol{v = -\omega\sqrt{A^2 - x^2}}$$

Negative sign shows that v has a direction opposite to the positive direction of $x - axis$.

Mention the position where the velocity of a particle executing SHM is maximum and minimum?

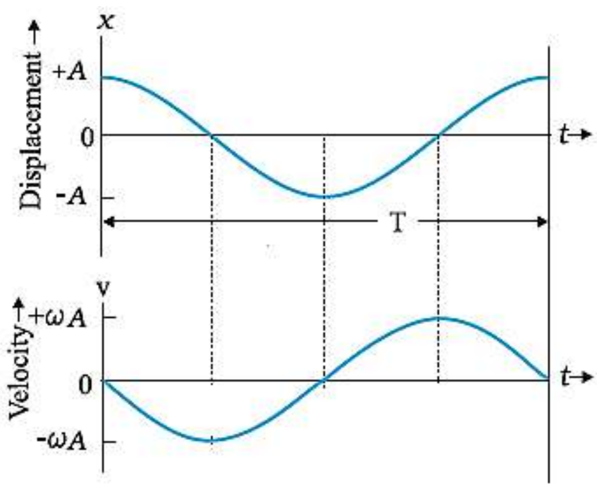
The equation $v = -\omega\sqrt{A^2 - x^2}$ tells that,

(i) When $x = 0$, $v = \omega A$ - velocity is maximum at equilibrium (mean) position

(ii) When $x = A$, $v = 0$ - velocity is minimum at extreme position.

What is the phase difference between velocity and displacement of a particle executing SHM? Represent it on a graph.

Phase difference is 90^0 or $\pi/_2$



Obtain the expression for Acceleration of the particle executing SHM.

We have $x = A\cos(\omega t + \phi)$

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$$

$$\boldsymbol{a = -\omega^2 x}$$

Negative sign indicates that the direction of displacement and acceleration are opposite to each other.

Mention the position where the acceleration of a particle executing SHM is maximum and minimum?

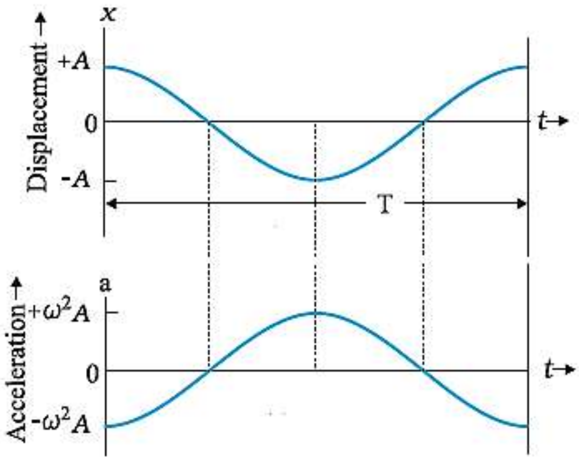
The equation $a = -\omega^2 x$ tells that,

(i) When $x = 0$, $a = 0$, acceleration is minimum at mean position.

(ii) when $x = A$, $|a| = \omega^2 A$, acceleration is maximum at extreme position.

What is the phase difference between acceleration and displacement of a particle executing SHM? Represent it on a graph.

Phase difference is 180^0 or π



Mention the characteristics of SHM.

(i) It is periodic motion.

(ii) It is to and fro motion about its mean position.

(iii) The acceleration is directly proportional to displacement.

(iv) The acceleration is always directed towards the mean position.

State Force law for SHM and hence obtain the expression for angular frequency.

In simple harmonic motion force is the proportional to the displacement of the particle and is directed towards the mean position.

Acceleration of a particle executing SHM is given by, $a = -\omega^2 x$

From Newton's second law, $F = ma$

$$F = m(-\omega^2 x)$$

$$F = -m\omega^2 x$$

$$\boldsymbol{F = -kx} \qquad where\ k = m\omega^2$$

Negative sign indicates that force and displacement are oppositely directed.

Now we have $k = m\omega^2$ implies $\boldsymbol{\omega = \sqrt{\frac{k}{m}}}$

A particle oscillating under a force given by $F = -kx$ is called linear harmonic oscillator

Derive the expression for kinetic energy and potential energy of a simple harmonic oscillator and hence find its total energy OR Obtain the expression for energy in SHM.

A particle executing SHM possess,

(i) Kinetic energy - because it is moving.

(ii) Potential energy - because it is subjected to conservative force $F = -kx$

Kinetic energy, $K = \frac{1}{2}mv^2$

$$K = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi)$$

$$K = \frac{1}{2}k\,A^2 \sin^2(\omega t + \phi)$$

Potential energy, $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}k\,A^2 \cos^2(\omega t + \phi)$$

Total energy, $E = K + U = \frac{1}{2}k\,A^2 \sin^2(\omega t + \phi) + \frac{1}{2}k\,A^2 \cos^2(\omega t + \phi)$

$$E = \frac{1}{2}k\,A^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$\boldsymbol{E = \frac{1}{2}k\,A^2}$$

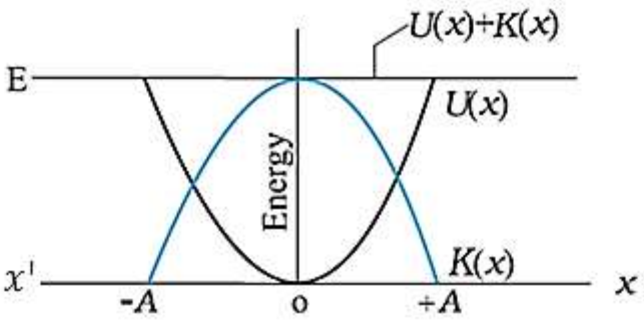
Explain the variation of kinetic and potential energies of a particle in SHM between zero and their maximum values with diagram giving total energy.

(i) When the particle is at mean position, $x = 0$, $U = 0$ and $K = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}k\,A^2$

At mean position kinetic energy is maximum and potential energy is zero.

(ii) When the particle is at extreme position, $x = A$, $U = \frac{1}{2}k\,A^2$ and $K = 0$

At the extreme positions kinetic energy is zero and potential energy is maximum.



Derive an expression for time period of oscillation of a mass attached to a spring OR Derive the expression for Time period of oscillating string.

Consider a block of mass m attached to a spring.

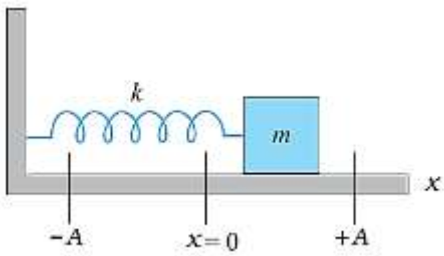
If the block is pulled and released, it executes to and fro motion.

Let $x = 0$ be the mean position of the block.

The restoring force of the block is given by, $F = -kx$

$$ma = -kx$$

$$a = -\left(\frac{k}{m}\right)x \quad ---(1)$$



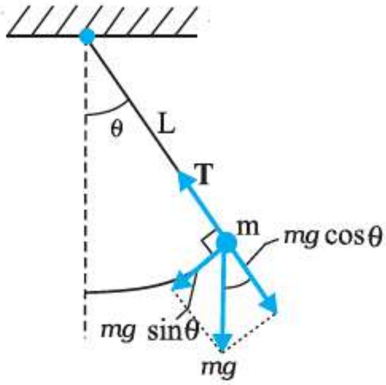
The standard equation for SHM is, $a = -\omega^2 x \quad ---(2)$

On comparing, $\omega^2 = \frac{k}{m}$

$$\omega = \sqrt{\frac{k}{m}}$$

Time period of the block is, $\boldsymbol{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$

Derive the expression for time period of Simple pendulum.



Consider a simple pendulum of mass m tied to a string of length L.
Let θ be the angle made by the string with the vertical.
The bob has two accelerations (i) radial acceleration
(ii) tangential acceleration
Radial acceleration provided by $T - mg\cos\theta$
Tangential acceleration provided by $mg\sin\theta$
Radial force gives zero torque.
Therefore, Torque on the bob $|\vec{\tau}| = |\vec{r} \times \vec{F}|$

$$\tau = -Lmg\sin\theta$$

Negative sign indicates that the restoring torque tends to reduce angular displacement.
By Newton's second law, $\tau = I\alpha$

$$I\alpha = -Lmg\sin\theta$$

$$\alpha = -\frac{mgL}{I}\sin\theta$$

If θ is small, $\sin\theta \approx \theta$, $\alpha = -\frac{mgL}{I}\theta$

Comparing with, $\alpha = -\omega^2\theta$

$$\omega^2 = \frac{mgL}{I}$$

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{mL^2}{mgL}} \qquad (I = mL^2)$$

$$\boldsymbol{T = 2\pi\sqrt{\frac{L}{g}}}$$

Illustration:

(i) What is the length of a simple pendulum, which ticks second?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Here $T = 2\ s$

$$L = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 2^2}{4 \times 3.14^2} = 0.99\ m \approx 1\ m$$

(ii) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall? and why?

Yes, the clock gives correct time during free fall. The motion in the wristwatch depends on spring action and has nothing to do with acceleration due to gravity.

CHAPTER-14 **WAVES**

Waves and Types of waves

What is a wave? Give examples.

A Wave is a sort of disturbance which transmits energy and momentum in a medium without transfer of particle.

Ex: Sound waves, light waves, waves of water, matter waves etc.

What is wave motion?

The propagation of disturbance from one point to another point is called wave motion.

Classify the types of waves based on propagation with examples.

(i) Mechanical waves: Waves which requires a material medium for their propagation are called mechanical waves.

Ex: Seismic waves, Water waves, sound waves etc.

(ii) Electromagnetic waves: Waves which do not require a material medium for their propagation are called mechanical waves.

Ex: Light waves, X-rays, micro waves etc.

(iii) Matter waves: Waves associated with moving material particles are called matter waves.

Louis de Broglie theoretically suggested that a moving matter such as electron, proton, neutron, atoms or molecules is associated with matter waves.

Mention the characteristics of mechanical waves.

1) They cannot propagate in vacuum.
2) Particles in the medium will vibrate.
3) Travels at relatively lower speed in a medium.
4) Propagation of mechanical wave depends on elastic properties of the medium.

Mention the characteristics of electromagnetic waves.

1) They can travel in vacuum
2) Electric and magnetic field will oscillate
3) Travels at relatively higher speed in a medium and speed of the wave is 299792458 m/s
4) The speed depends on Permittivity and permeability of the medium

Classify the types of waves based on vibration of particles of the medium with examples.

(i) Transverse waves: If the oscillations/vibrations in the medium are perpendicular to the direction of wave propagation, then the waves are called transverse waves.

Ex: Light waves, Seismic S-waves etc.

(ii) Longitudinal waves: If the oscillations/vibrations in the medium are along or parallel to the direction of wave propagation, then the waves are called transverse waves.

Ex: Sound waves, pressure waves, Seismic P-waves etc.

Mention the characteristics of transverse waves.

1) They contain alternate crests and troughs.
2) Crests are the elevations formed in the medium and troughs are the depression formed in a medium.

3) Transverse waves are either mechanical or electromagnetic.
4) They can travel only in solids, if waves are mechanical.
5) These waves can be polarized.

Mention the characteristics of longitudinal waves.
1) They contain alternate compressions and rarefactions.
2) Compressions are the portion of the medium having more density and rarefactions are the portions of the medium having less density.
3) They are always mechanical.
4) They can travel in solids, liquids and gases.
5) These waves cannot be polarized.

Terms related to Waves

What s amplitude?
The maximum displacement of a particle of the medium from its equilibrium position is known as amplitude.

What is time period?
It is the time taken by the particle to complete one oscillation **OR**
It is the time in which one wave is set up in a medium.

What is frequency? Mention its SI unit.
It is the number of waves setup in a medium in one second.

$$Frequency = \frac{Number\ of\ waves}{time}$$

Frequency is expressed in hertz (Hz) and 1 hertz=1 cycle per second.

What is wave length? Give its SI unit.
The distance between two consecutive particles which are in same phase.
Wave length is expressed in metre.

What is a progressive wave?
A wave which travels from one point to another point of the medium in the same direction without change in its amplitude is known as a progressive wave.

What is progressive wave equation?
The equation $y(x,t) = a\sin(kx - \omega t + \phi)$ which gives the displacement of a particle in a medium at any point, at any instant of time is called a progressive wave equation.

Give the equation for a progressive wave and explain the terms.
A sinusoidal wave travelling in negative x direction is given by, $y(x,t) = a\sin(kx - \omega t + \phi)$

$y(x,t)$ = Displacement of the particle
a = Amplitude of the wave
k = Angular wave number or propagation constant
ω = angular frequency of wave
ϕ = initial phase angle

Keep in mind

Other forms of wave equations travelling in negative x-direction.

$y = \text{a} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$y = \text{a} \sin \omega \left(\frac{x}{v} - t\right)$

$y = \text{a} \sin \frac{2\pi}{\lambda} (x - vt)$

The above equation are obtained by substituting,

$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{v}$ and $\omega = 2\pi\nu = \frac{2\pi}{T}$

Define Phase of a vibrating particle. Explain.

The state of vibration of a particle at that instant of time with reference to its equilibrium position is called phase.
The quantity $(kx - \omega t + \phi)$ represents the phase of the vibrating particle.
At $x = 0$ and $t = 0$, $(kx - \omega t + \phi) = \phi$ is called initial phase angle.

Define angular wave number or propagation constant.

It is the number of waves that can be accommodated per unit length **OR**

It is the change in phase with respect to distance.

How is propagation constant related to wavelength of a wave?

It is given by $k = {}^{2\pi}/_{\lambda}$ and expressed in rad/metre.

Define Angular frequency.

It is given by $\omega = {}^{2\pi}/_{T}$ and expressed in rad/second.

Define wave velocity.

The distance travelled by the wave in one second is called wave velocity.

Mention the expression for Speed of a travelling wave.

Speed of a wave is given by, $v = \lambda f$ or $v = {}^{\lambda}/_{T}$ or $v = {}^{\omega}/_{k}$

This is the general relation for all progressive waves.

Name the quantities associated with a wave,
(i) that changes when a wave travels from one medium to another.
(ii) that remains unchanged when a wave travel from one medium to another.

When a wave enters from one medium to another, (i) both speed and wavelength change but (ii) the frequency remain unchanged.

Illustration:

Calculate the wavelength of a wave whose angular wave number is 10π radian per metre?

$k = {}^{2\pi}/_{\lambda} \Rightarrow \lambda = \frac{2\pi}{10\pi} = \frac{1}{5} = 0.2\ m$

Calculate the period of a wave of wavelength 0.005 m which travels with a speed of 50 $cm - s^{-1}$.

$$v = \frac{\lambda}{T}$$

$$T = \frac{\lambda}{v} = \frac{0.005}{50 \times 10^{-2}} = \frac{0.5}{50} = \frac{0.1}{10} = 0.01\ s$$

Speed of travelling wave

Mention the expression for speed of a transverse wave on stretched string.

It is given by, $\boldsymbol{v = \sqrt{T/\mu}}$

Where $T \rightarrow$ Tension in the string and $\mu \rightarrow$ mass per unit length of the string.

Mention the expression for speed of sound (Longitudinal wave).

It is given by, $\boldsymbol{v = \sqrt{E/\rho}}$

Where $E \rightarrow$ Modulus of elasticity and $\rho \rightarrow$ density of the medium

(i) In solids, $E = Y$ $(youngs\ modulus)$ then $\boldsymbol{v = \sqrt{Y/\rho}}$

(ii) In fluids, $E = B$ $(Bulk\ modulus)$ then $\boldsymbol{v = \sqrt{B/\rho}}$

Keep in mind

- Waves setup in a stretched string are mechanical waves.
- The speed of a mechanical wave is determined by the restoring force setup in the medium when it is disturbed and the inertial properties (mass density) of the medium.
- Liquids and solids generally have higher speed of sound than gases. Because they are much more difficult to compress than gases and so have much higher values of bulk modulus.

Obtain the Newton's formula for speed of sound in a gas.

Newton suggested that,

(i) During compression, temperature of the medium increases and heat is lost to the surrounding.

(ii) During rarefaction, temperature is absorbed from the surrounding.

Thus the conditions are, isothermal but there is a change in pressure and volume of the air.

$$PV = RT$$

$$\therefore PV = (P + \Delta P)(V + \Delta V)$$

$$PV = PV + P\Delta V + V\Delta P \qquad \text{Since } \Delta P\Delta V \approx 0$$

$$P\Delta V = -V\Delta P$$

$$P = -\frac{\Delta P}{\left(\Delta V/V\right)} = B$$

But, $v = \sqrt{B/\rho}$ in air. Then, $\boldsymbol{v = \sqrt{P/\rho}}$

According to this formula, at STP, $v = 280\ m/s \neq 330\ m/s$

Discuss the Laplace correction and arrive at the formula modified by him.

Laplace suggested that,

(i) Vibrations of the layers of air are so rapid and no time for heat transfer between the layers.

(ii) Air is a bad conductor of heat.

Thus, the condition is Adiabatic but not isothermal.

$PV^{\gamma} = Constant$ where γ →ratio of specific heat

Differentiate, $P(\gamma V^{\gamma} dV) + V^{\gamma} dP = 0$

$$P\,\gamma = -\frac{V^{\gamma}\, dP}{V^{\gamma-1}\, dV}$$

$$P\,\gamma = -\frac{dP}{\left({dV}/{V}\right)} = B$$

$$\boldsymbol{v = \sqrt{{\gamma P}/{\rho}}}$$

This Formula is called Newton's-Laplace formula.

According to this formula, at STP, $v = 332\ m/s\ \approx 331.3\ m/s$, a good agreement with practical value.

Superposition of waves

State the principle of superposition of waves.

When two pulses of equal and opposite shapes move towards each other and overlap, the resultant displacement is the algebraic sum of the displacement due to each pulse. This is known as the principle of superposition of waves.

Give the theory of principle of superposition of waves.

Consider two harmonic travelling waves on a stretched string, both with the same ω (angular frequency) and k (wave number).

Let us further assume that their amplitudes are equal and they are both travelling in the positive direction of x-axis.

$y_1 = a\sin(kx - \omega t)$

$y_2 = a\sin(kx - \omega t + \phi)$

The net displacement is, $y = y_1 + y_2 = a\sin(kx - \omega t) + a\sin(kx - \omega t + \phi)$

$$y = a\left\{2\sin\left(\frac{kx - \omega t + kx - \omega t + \phi}{2}\right)\cos\left[\frac{kx - \omega t - (kx - \omega t + \phi)}{2}\right]\right\}$$

$$y = 2a\cos\frac{\phi}{2}\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

$$\boldsymbol{y = A\sin\left(kx - \omega t + \frac{\phi}{2}\right)} \qquad where\ A = 2a\cos\frac{\phi}{2}$$

The above equation represents harmonic travelling wave in the positive direction of x-axis, with the same frequency and wavelength.

Keep in mind

- When the waves are in phase, $\phi = 0$, then $A = 2a$ and $y = 2a\sin(kx - \omega t)$
  The resultant wave has amplitude $2a$, the largest possible value for A.
  This refers to the constructive interference of the two waves where the amplitudes add up in the resultant wave.
- For $= \pi$, the waves are completely out of phase. then $A = 0$ and $y = 0$
  This refers to the destructive interference where the amplitudes subtract out in the resultant wave.

Reflection of waves

What is reflection of waves?
When a wave travelling in a medium, meets a rigid boundary, it gets returned to the same medium is called reflection of wave.

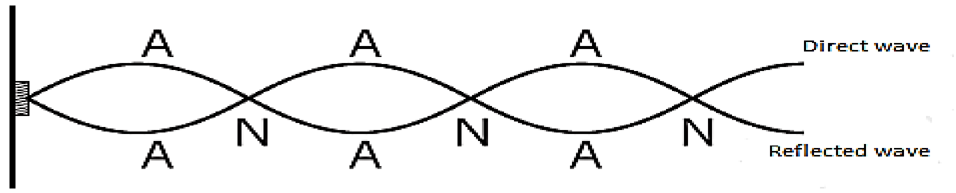
What is the shape of the reflected wave?
The reflected wave has same shape as the incident wave.

What is the phase angle between the incident wave and the wave reflected at a rigid boundary?
The reflected suffers a phase change of π or 180^0

What is standing wave or stationary wave? Give examples.
When two waves of equal amplitude and wavelength travelling along a line in opposite direction and superimpose the resulting wave pattern is called a stationary wave of standing wave.
Ex: (i) Stationary waves produced in the closed and open pipe.
(ii) Stationary waves produced in the vibrating string.

What are nodes and anti-nodes in stationary waves? Show them diagrammatically.
Nodes: At some positions in the wave, the amplitude of the particles are zero, these points are called Nodes.

Antinodes: At some points in a wave, the amplitude of the particles are maximum or the particles vibrate with maximum amplitude, these points are called antinodes.



Mention the conditions for nodes and antinodes.

(1)Node: For minimum amplitude, $\textbf{Length} = \boldsymbol{n}\frac{\boldsymbol{\lambda}}{\mathbf{2}}$ where $n = 0,1,2, \dots \dots .$

(2)Antinode: For maximum Amplitude, $\textbf{Length} = \left(\boldsymbol{n} + \frac{\mathbf{1}}{\mathbf{2}}\right)\frac{\boldsymbol{\lambda}}{\mathbf{2}}$ where $n = 0,1,2, \dots \dots .$

What is distance between (i) two consecutive node or antinode and (ii) a node and the antinode?
(i) The distance between two consecutive node or antinode is $\lambda/_2$
(ii) The distance between a node and the antinode is $\lambda/_4$

At which positions (or) locations of the stationary wave, the pressure changes are maximum and minimum?
Pressure changes are maximum at Node and pressure changes are minimum at antinode.

At which positions (or) location of the stationary wave, the displacement is maximum and minimum?
Displacement is maximum antinode and displacement is minimum at node.

What are normal modes of oscillation in a stationary wave?

In a stationary wave, the possible frequencies of oscillation of the system is characterised by set of natural frequencies called Normal modes.

What are harmonics in a stationary wave?

For a vibrating system the frequencies which are integral multiples of fundamental frequency are called harmonics.

What are overtones in a stationary wave?

For a vibrating system, frequencies greater than fundamental frequencies are called overtones.

What is the meaning of the fundamental mode (or) first harmonic of oscillation in a stationary wave?

In a stationary wave, the oscillation of the system with lowest possible natural frequency is called as fundamental frequency (or) first harmonic.

Give the differences between progressive and stationary waves.

Progressive wave	Stationary wave
The wave travel continuously with certain velocity	The waves do not move. It remains localized.
The propagation of the disturbance from particle due to elastic properties of the medium give rise to a progressive wave	The superposition of two identical waves traveling in opposite direction along the same line results in a stationary wave
Amplitude of vibration is the same for every particle of the medium along the wave	The amplitude of vibration varies from zero at node & maximum at antinode
No particles in the medium is completely at rest	The particles at node are permanently at rest
There is a net transfer of energy in the direction of propagation of wave	There is no net transfer of energy across any section of the medium
The wave equation is of the form $y(x,t) = A\,sin(\omega t - kx)$	The wave equation is of the form $y(x,t) = 2A\,cos\,(kx)\,sin\,\omega t.$

Discuss different modes of vibration (first three harmonics) produced in a closed pipe.

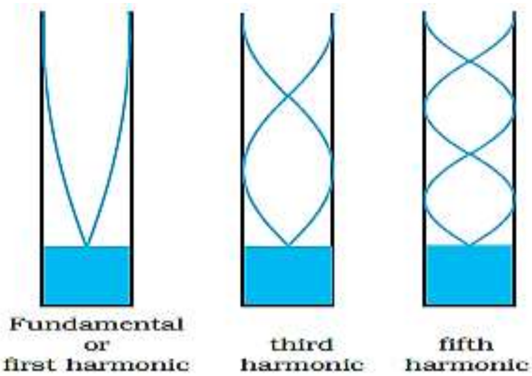
A closed pipe is one in which one end is closed and other is opened.

Inside a closed pipe,

(i) Open end always has an antinode, because open end have maximum freedom to vibrate.

(ii) Closed end always has a node, because closed end are not free.

(iii) Frequency of vibration of air column depends on (a) length of pipe and (b) mode of vibration.



Fundamental or First harmonic: If the air column consists of single node and a single antinode, the mode of vibration is called Fundamental or First harmonic.

If λ_1 is the wavelength and L is the length of the pipe, then

$$L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

f_1 is called fundamental frequency and this is the minimum frequency.

Third harmonic: In second mode of vibration two modes and two antinodes are formed.

If λ_2 is the wavelength and L is the length of the pipe, then

$$L = \frac{3\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4L}{3}$$

$$f_2 = \frac{v}{\lambda_2} = 3\left(\frac{v}{4L}\right) = 3f_1$$

Fifth harmonic: In this type of vibration three nodes and three antinodes are formed.

If λ_3 is the wavelength and L is the length of the pipe, then

$$L = \frac{5\lambda_3}{4} \Rightarrow \lambda_3 = \frac{5L}{3}$$

$$f_3 = \frac{v}{\lambda_3} = 5\left(\frac{v}{4L}\right) = 5f_1$$

Now, $\boldsymbol{f_1 : f_2 : f_3 = 1 : 3 : 5}$, hence Only odd harmonics are present.

Illustration:

The distance between a node and next antinode in a stationary wave pattern is 0.08m. What is the wavelength of the wave?

The distance between a node and the antinode is $\lambda/4$

$$\frac{\lambda}{4} = 0.08 \Longrightarrow \lambda = 0.32\, m$$

For what wavelength of waves, does a closed pipe of length 30 cm emit the first overtone?

$$f_2 = \frac{v}{\lambda_2} = 3\left(\frac{v}{4L}\right)$$

$$\frac{v}{\lambda_2} = \frac{3 \times v}{4 \times 30} = \frac{v}{40}$$

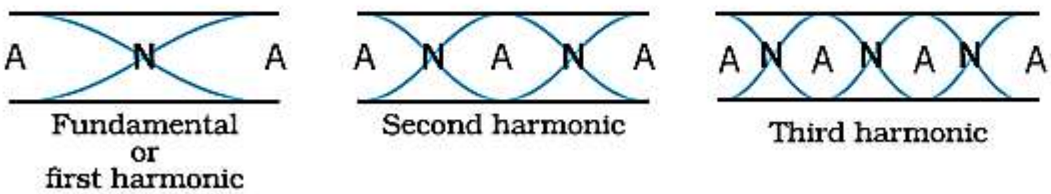
$$v = 40\, m$$

The fundamental frequency of a closed pipe is 80Hz. What is the frequency of first overtone?

$$f_2 = 3f_1$$

$$f_2 = 3 \times 80 = 240\, Hz$$

Discuss different modes of vibration (first three harmonics) produced in an open pipe.



A pipe opened at both ends is called open pipe. In open pipe antinodes are always formed at the open ends.

Fundamental or first harmonic: If the air column vibrate such that the entire air column consists of a single node and two antinodes, the mode of vibration is called first mode of vibration of fundamental mode.

If λ_1 is the wavelength and L is the length of the pipe, then

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

f_1 is called fundamental frequency and this is the minimum frequency.

Second harmonic: This mode of vibration consists of two nodes and three antinodes.

If λ_2 is the wavelength and L is the length of the pipe, then

$$L = \lambda_2$$

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2f_1$$

Third harmonic: This mode of vibration consists of three nodes and four antinodes.

If λ_3 is the wavelength and L is the length of the pipe, then

$$L = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = 3f_1$$

Now, $\boldsymbol{f_1 : f_2 : f_3 = 1 : 2 : 3}$, hence both even and odd harmonics are present.

Illustration:

When is the fundamental frequency of the sound emitted by a closed pipe is same as that emitted by an open pipe?

$$(f_1)_{closed} = \frac{v}{4L_c}$$

$$(f_1)_{open} = \frac{v}{2L_o}$$

$$\frac{v}{4L_c} = \frac{v}{2L_o} \Rightarrow \frac{1}{2L_c} = \frac{1}{L_o}$$

$$L_o = 2L_c \text{ or } L_c = \frac{L_o}{2}$$

A closed pipe and open pipe have same frequency for the first overtone. What is the ratio of their lengths?

$$(f_2)_{closed} = \frac{3v}{4L_c}$$

$$(f_2)_{open} = \frac{v}{L_o}$$

$$\frac{3v}{4L_c} = \frac{v}{L_o} \Rightarrow \frac{3}{4L_c} = \frac{1}{L_o}$$

$$4L_c = 3L_o \text{ or } \frac{L_c}{L_o} = \frac{3}{4}$$

The second overtone of closed pipe of length 1 m is in unison with the third overtone of an open pipe. What is the length of the open pipe?

$$(f_3)_{closed} = 5\left(\frac{v}{4L_c}\right) = \frac{5v}{4}$$

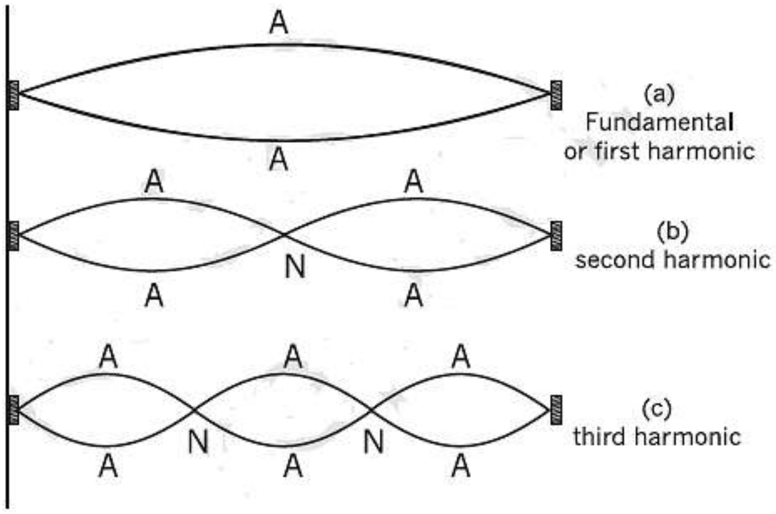
$$(f_4)_{open} = 4\left(\frac{v}{2L_o}\right)$$

$$\frac{5v}{4} = \frac{4v}{2L_o} \Rightarrow \frac{5}{4} = \frac{2}{L_o}$$

$$L_o = \frac{8}{5} = 1.6\,m$$

Discuss different modes of vibration on a stretched string.

In a stretched string node are always formed at the fixed ends.



Fundamental or first harmonic: Two nodes and a single antinode are formed.

If λ_1 is the wavelength and L is the length of the string, then

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

f_1 is called fundamental frequency and this is the minimum frequency.

Second harmonic: This mode of vibration consists of three nodes and two antinodes.

If λ_2 is the wavelength and L is the length of the string, then

$$L = \lambda_2$$

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2f_1$$

Third harmonic: This mode of vibration consists of four nodes and three antinodes.

If λ_3 is the wavelength and L is the length of the string, then

$$L = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = 3f_1$$

Now, $\boldsymbol{f_1 : f_2 : f_3 = 1 : 2 : 3}$, hence both even and odd harmonics are present.

Obtain the expression for frequency of vibration of stretched string.

Speed of transverse wave is given by, $v = \sqrt{T/\mu}$

(i)Frequency, $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{1}{2L}\sqrt{T/\mu}$

(ii)Frequency, $f_2 = \frac{v}{\lambda_2} = 2\left(\frac{v}{2L}\right) = \frac{2}{2L}\sqrt{T/\mu}$

(iii)Frequency, $f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right) = \frac{3}{2L}\sqrt{T/\mu}$

Generally, $\boldsymbol{f = \frac{n}{2L}\sqrt{T/\mu}}$

State the laws of vibration in a stretched string

(1) Law of length: Frequency of vibrating string is inversely proportional to length of the string.

$$f \propto \frac{1}{L}$$

(2) Law of tension: Frequency of vibrating string is directly proportional to square root of the tension in the string.

$$f \propto \sqrt{T}$$

(3) Law of mass: Frequency of vibrating string is inversely proportional to square root of the mass per unit length of the string.

$$f \propto \frac{1}{\sqrt{\mu}}$$

Beats

What are beats?

The periodic waxing (increase or rise) and waning (decrease or fall) in the intensity of sound due to superposition of two sound waves of nearly same frequencies traveling in same direction are called beats.

This is a phenomena based on the principle of superposition of waves.

What is beat period?

The time interval between two consecutive waxing or waning is called as beat period.

Give the theory of beats.

Consider two sound waves of same amplitude (a) and nearly equal angular frequencies $\omega_1$and ω_2 such that $\omega_1 > \omega_2$

Let two waves can be represented as $s_1 = a\cos\omega_1 t$ and $s_2 = a\cos\omega_2 t$

The resultant displacement is given by, $s = s_1 + s_2 = a\cos\omega_1 t + a\cos\omega_2 t$

$s = a(\cos\omega_1 t + \cos\omega_2 t)$

$s = 2\,\mathrm{a}\cos\left[\frac{(\omega_1 - \omega_2)t}{2}\right]\cos\left[\frac{(\omega_1 + \omega_2)t}{2}\right]$

$\boldsymbol{s = 2\,a\cos\omega_b t\cos\omega_a t}$ $\quad$ $where\ \omega_a = \frac{\omega_1 + \omega_2}{2}\ and\ \omega_b = \frac{\omega_1 - \omega_2}{2}$

This equation is similar to $\boldsymbol{s = R\cos\omega t}$

Here $\boldsymbol{R = 2\,a\cos\omega_b t}$ is the amplitude of the resultant wave and $\boldsymbol{\omega = \omega_a}$ is the average angular frequency of the resultant wave.

The amplitude and hence intensity of resultant wave is maximum when $\boldsymbol{\cos\omega_b t = \pm 1}$

Intensity of the resultant wave waxes and wanes with a frequency which is $\boldsymbol{2\,\omega_b = \omega_1 - \omega_2}$

Since $\omega = 2\pi\nu$, the beat frequency is given by $\boldsymbol{\nu_{beat} = \nu_1 - \nu_2}$

Mention the applications of beats.

The phenomenon of beats can be used to,

- determining the unknown frequency of a tuning fork using fork of known frequency.
- tune musical instruments.
- detection of harmful gases in mines.

I PUC PHYSICS(33) BLUEPRINT FOR MODEL QUESTION PAPER 2023-24

QP Part	Question type	Number of questions to be set	Marks alloted	Number of questions to be answered	Marks alloted
A	MCQ	15	15	15	15
	Fill in the blank(FIB)	05	05	05	05
B	SA (2 Marks)	09	18	05	10
C	SA (3 Marks)	09	27	05	15
D	LA (5 Marks)	06	30	03	15
	Numerical Problem(NP) (5 Marks)	04	20	02	10
	Total	**48**	**115**	**30**	**70**

Unit	Sr. No	Chapter/ Content domain/ Unit/ Theme	No. of periods	Marks	Remember(41 marks)					Understand(33 marks)				Apply(23 marks)				HOTS(18 marks)		
					MCQ 1 mark	FIB 1 Mark	SA 2 Marks	SA 3 Marks	LA	SA 2 Marks	NP 2 Marks	SA 3 Marks	LA 5 mark	SA 2 Marks	NP 3 Marks	LA 5 mark	NP 5 mark	MCQ 1 mark	SA	NP 5 mark
I	1	Units and measurement	3	3	1		1													
	2	Motion in a straight line	7	6												1		1		
II	3	Motion in a plane	13	12	1	1		1			1					1				
III	4	Laws of motion	13	12	2		1					1								1
IV	5	Work energy and power	12	11	1			1		1						1				
V	6	System of particles and rigid body	11	11			1						1(ST)		1			1		
VI	7	Gravitation	10	9	1	1				1										1
VII	8	Mechanical properties of solids	4	4	1							1								
	9	Mechanical properties of fluids	5	5	1	1		1												
	10	Thermal properties of matter	11	11	1		1					1					1			
VIII	11	Thermodynamics	8	8			1						1(ST)					1		
IX	12	Kinetic theory	5	5	1	1						1								
X	13	Oscillations	8	8	1		1													1
	14	Waves	10	10	1	1		1					1							
		Total	**120**	**115**	**12**	**5**	**12**	**12**	**-**	**4**	**2**	**12**	**15**	**-**	**3**	**15**	**5**	**3**	**-**	**15**

ST => SPLIT TYPE QUESTION (1+2+2) OR (2+3) OR (1+4) OR (1+1+3)

Weightage to objectives:

Objectives	Weightage	Marks
Knowledge	35%	41
Understanding	29%	33
Application	20%	23
HOTS	16%	18

Weightage to level of difficulty:

Level	Weightage	Marks
Easy	40%	46
Average	40%	46
Difficult	20%	23

GENERAL GUIDELINES FOR SETTING THE QUESTION PAPER

1. Variation of 1 mark in each chapter or unit weightage is permitted while preparing the blue print and the total marks should not exceed 115.
2. The question paper should be prepared on the basis of blueprint following the weightage of marks fixed for each chapter. The questions must be framed to check the specific cognitive level as mentioned in the blueprint.
3. Questions should be clear, unambiguous, understandable and free from grammatical errors.
4. Questions which are based on same concept, law, fact etc. and which generate the same answer should not be repeated under different forms (MCQ, FIB, VSA, LA and NP).
5. The answers for the questions should be available in the prescribed text book or can be derived from the concepts of text book for application/reasoning/analytical/HOTS questions.
6. When a question carrying 3 or 5 marks is split, the sub questions should be derived from the same concept or different concepts of same chapter.
7. Only one 5 mark numerical problem has to be set from chapters corresponding to a pair of consecutive units like I & II, III & IV, V & VI, VII & VIII, IX & X.
8. In part A (I main) 3 MCQ and in part D (VI main) 3 numerical problems of same difficulty level must be framed to check Higher Order Thinking Skills.
9. Only one simple numerical problem can be included in each of the part B (2 mark) and part C (3 mark).

&&&&&&&

MODEL QUESTION PAPER 2023-24

I PUC - PHYSICS (33)

Time: 3 hours 15 min. **Max Marks: 70**

General Instructions:

1. *All parts are compulsory.*
2. *For Part – A questions, first written-answer will be considered for awarding marks.*
3. *Answers without relevant diagram / figure / circuit wherever necessary will not carry any marks.*
4. *Direct answers to the numerical problems without detailed solutions will not carry any marks.*

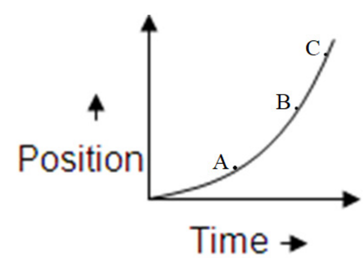
PART – A

I. Pick the correct option among the four given options for ALL of the following questions: **15 × 1 = 15**

1. The number of significant figures in 3.500 is

(A) 2 (B) 3 (C) 4 (D) 1

2. If v_A, v_B and v_C are the magnitudes of instantaneous velocities corresponding to the points A, B and C of the given position-time graph of a particle respectively, then



(A) $v_A = v_B = v_C$ (B) $v_A > v_B > v_C$

(C) $v_A < v_B < v_C$ (D) $v_A < v_B = v_C$

3. A vector is multiplied with a positive integer. The direction of the resultant vector is

(A) same as the initial vector (B) opposite to the initial vector

(C) perpendicular to the initial vector (D) not specified

4. 'An external force is required to keep a body in motion'. This is the statement of

(A) Newton's first law of motion (B) Newton's second law of motion

(C) Aristotelian law of motion (D) Newton's third law of motion

5. The non-contact force encountered in mechanics is

(A) normal reaction (B) frictional force (C) tension in a spring (D) gravitational force

6. 1 calorie is equal to

(A) 4.186 J (B) 1.6×10^{-19} J (C) 3.6×10^{6} J (D) 10^{-7} J

7. A girl is sitting with folded hands on a swivel chair rotating with considerable angular speed. 'I' and 'ω' are the moment of inertia and angular speed of the chair along with girl about the axis of rotation. She stretches her arms horizontally while the chair is rotating. During this

(A) I decreases and ω increases (B) I increases and ω decreases

(C) both I and ω increase (D) both I and ω decrease

8. The SI unit of universal gravitational constant (G) is :

(A) $N\,m^2\,kg^{-1}$ (B) $N\,m^2\,kg^{-2}$ (C) $N\,m\,kg^{-2}$ (D) $N\,m^2\,kg^{-3}$

9. The ratio of lateral strain to longitudinal strain in a stretched wire is called

(A) shear strain (B) compressibility (C) Poisson's ratio (D) Young's modulus

10. The angle of contact is acute in case of

(A) water-lotus leaf interface (B) water-waxy surface interface

(C) water-oily surface interface (D) water-glass interface

11. The change from solid state to vapour state without passing through the liquid state is called

(A) vaporisation (B) melting (C) regelation (D) sublimation

12. Below are two statements:

(I) In a cyclic process, the total heat absorbed equals the work done by the system.

(II) The change in internal energy of the system is zero during cyclic process.

(A) Statement I is wrong but the statement II is correct

(B) Statement I is correct but the statement II is wrong

(C) Both the statements I and II are correct and II is the correct explanation for I

(D) Both the statements I and II are correct and II is not the correct explanation for I

13. The mean free path for gas molecules is given by the expression (with symbols having their usual meaning)

(A) $l = \frac{\pi}{\sqrt{2} n d^2}$ (B) $l = \frac{1}{\sqrt{2} n d^2}$ (C) $l = \frac{1}{\sqrt{2} \pi n d^2}$ (D) $l = \frac{\sqrt{2}}{\pi n d^2}$

14. The function of time which is not periodic among the following is

(A) $\sin \omega t$ (B) $\cos \omega t$ (C) $e^{-\omega t}$ (D) $\sin \omega t + \cos \omega t$

15. Air column present in an open pipe is vibrating in fundamental mode. It contains

(A) a node and an antinode (B) a node and two antinodes

(C) two nodes and an antinode (D) two nodes and two antinodes

II. Fill in the blanks by choosing appropriate answer given in the brackets for ALL the following questions: $5 \times 1 = 5$

(decrease, elastic, elliptical, beats, increase, speed)

16. During uniform circular motion, an object moves in circular path with constant___________.

17. According to Kepler, all planets move in ______________ orbits around the Sun.

18. The viscosity of liquids decreases with _____________in temperature.

19. According to kinetic theory of gases, collisions between the molecules of a gas are ______________.

20. The phenomenon often used by artists to tune their musical instruments is ______________.

PART – B

III. Answer any FIVE of the following questions: **5 × 2 = 10**

21. Mention any two applications of dimensional analysis.

22. Two vectors of same units have magnitude of 8 unit and 5 unit. What are the maximum and minimum magnitude of resultant that can be obtained with the two vectors?

23. Give any two methods of reducing friction.

24. State and explain work-energy theorem for a constant force.

25. Mention the two conditions required for the mechanical equilibrium of a rigid body.

26. Why does moon has no atmosphere? Explain.

27. What is meant by thermal expansion? Give relation between coefficient of volume expansion and coefficient of linear expansion of a material.

28. What is a Carnot engine? Name the working substance used in it.

29. Give the positions at which the potential energy of a particle executing SHM will be (i) maximum and (ii) zero.

PART – C

IV. Answer any FIVE of the following questions: **5 × 3 = 15**

30. What is meant by range of a projectile? Give the expression for the same. What is the angle of projection for which the range of a projectile maximum?

31. State and explain Newton’s second law of motion. Define SI unit of force.

32. Define work done by a force. Mention two cases in which work done by a force on an object is zero.

33. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 s. Calculate the angular acceleration of the wheel assuming the acceleration to be uniform.

34. Distinguish between the three different moduli of elasticity of a material.

35. State Bernoulli’s principle. Give Bernoulli’s equation in fluid dynamics. What is Magnus effect?

36. Explain briefly the land breeze.

37. Using the expression of total internal energy of one mole of monatomic gas, obtain the expression for the molar specific heat of a monatomic gas at constant volume.

38. Define (i) frequency and (ii) wavelength of a wave. Give an example for non-mechanical wave.

PART – D

V. Answer any THREE of the following questions: **3 × 5 = 15**

39. Derive the equation $x = v_0 t + \frac{1}{2}at^2$ using v-t graph.

40. Show that the trajectory of a projectile is a parabola.

41. Obtain the expressions for final velocities of two particles undergoing completely elastic collision in one-dimension considering second body to be initially at rest.

42. a) What is a rigid body? What type of motion is observed in a rigid body which is pivoted at the centre of mass? (2)

b) Prove that the time rate of change of the angular momentum of a particle is equal to the torque acting on it. (3)

43. a) State and explain first law of thermodynamics. (2)

b) Mention any three differences between isothermal and adiabatic processes. (3)

44. Write Newton's formula for speed of sound in air. Explain why and how Laplace modified Newton's formula for speed of sound.

VI. Answer any TWO of the following questions: $2 \times 5 = 10$

45. A body of mass of 8 kg is suspended by a rope of length 2.5 m from the ceiling. A force of 60 N is applied on the body in the horizontal direction. Find the angle that the rope makes with the vertical in equilibrium. (Take g = 10 ms^{-2}). Neglect the mass of the rope.

46. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh at a depth equal to half the radius of the earth if it weighs 250 N on the surface of earth? What will the weight of the same body at the centre of the earth?

47. A copper plate of mass 2 kg is heated to a temperature of 600^oC and then placed on large ice block at 0^oC. Calculate (i) the maximum quantity of heat that the copper plate can transfer to ice block and ii) the maximum amount of ice it can melt. Given: Specific heat of capacity of copper = 390 J kg^{-1} K^{-1} and latent heat fusion of water = 333×10^3 J kg^{-1}.

48. A block of mass 1 kg is fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0 and is released.

Calculate (i) angular frequency of oscillations of the block and (ii) the maximum speed with which the block crosses the mean position.

&&&&