Numerical Problems:

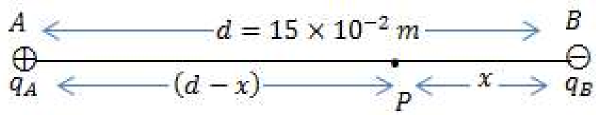
1) **Two charges $3 \times 10^{-8}C$ and $-2 \times 10^{-8}C$ are located $15cm$ apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. (Jul 2014)**

Given: $q_A = 3 \times 10^{-8}C$

$q_B = -2 \times 10^{-8}C$

$2a = d = 15\ cm = 15 \times 10^{-2}\ m$

(i) Let P be a point of zero potential anywhere in between A and B, which is at a distance x from q_B



Potential at P is V_P is given by, $V_P = V_A + V_B$

$V_A + V_B = 0 \qquad (V_P = 0)$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(d-x)} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{(d-x)} + \frac{q_B}{x}\right] = 0$$

$$\frac{q_A}{(d-x)} + \frac{q_B}{x} = 0 \qquad \left(since\ \frac{1}{4\pi\varepsilon_0} \neq 0\right)$$

$$\frac{3 \times 10^{-8}}{(15 \times 10^{-2} - x)} + \frac{-2 \times 10^{-8}}{x} = 0$$

$$\frac{3 \times 10^{-8}}{(15 \times 10^{-2} - x)} = \frac{2 \times 10^{-8}}{x}$$

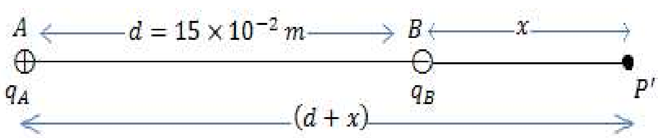
$$\frac{3}{(15 \times 10^{-2} - x)} = \frac{2}{x}$$

$3x = 30 \times 10^{-2} - 2x$

$3x + 2x = 30 \times 10^{-2} \Longrightarrow 5x = 30 \times 10^{-2}$

$x = 6 \times 10^{-2}\ m = 6\ cm$

(ii) Let P' be a point of zero potential anywhere outside A and B, which is at a distance x from q_B



$$\frac{q_A}{(d+x)} + \frac{q_B}{x} = 0 \qquad \left(since\ \frac{1}{4\pi\varepsilon_0} \neq 0\right)$$

$$\frac{3 \times 10^{-8}}{(15 \times 10^{-2} + x)} + \frac{-2 \times 10^{-8}}{x} = 0$$

$$\frac{3 \times 10^{-8}}{(15 \times 10^{-2} + x)} = \frac{2 \times 10^{-8}}{x}$$

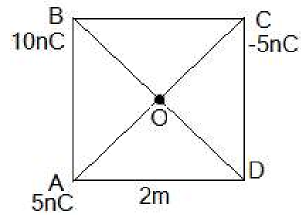
$$\frac{3}{(15 \times 10^{-2} + x)} = \frac{2}{x}$$

$3x = 30 \times 10^{-2} + 2x$

$3x - 2x = 30 \times 10^{-2} \Longrightarrow x = 30 \times 10^{-2}\ m$

$x = 30\ cm$

2) **ABCD is a square of side $2m$. Charges of $5nC, +10nC$ and $-5nC$ are placed at corners A, B and C respectively. What is the work done in transferring a charge of $5\mu C$ from D to the point of intersection of the diagonals? (Jul 2015)**



Given: $q_A = 5\,n\,C = 5 \times 10^{-9}\,C$
$q_B = 10\,n\,C = 10 \times 10^{-9}\,C$
$q_C = -5\,n\,C = -5 \times 10^{-9}\,C$
$Length, l = 2\,m$
$Test\ charge, q = 5\,\mu\,C = 5 \times 10^{-6}\,C$

We have, $AD = CD = 2\,m,\ BD = 2\sqrt{2}\,m$ and $AO = BO = CO = \sqrt{2}\,m$

Work done in moving $5\mu C$ charge from D to O is given by, $W = (P.d. bet\underline{n}. O\ and\ D) \times q$

Potential at O is given by, $V_O = V_{AO} + V_{BO} + V_{CO}$

$$V_O = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{AO} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{BO} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{CO}$$

$$V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AO} + \frac{q_B}{BO} + \frac{q_C}{CO}\right)$$

$$V_O = 9 \times 10^9\left(\frac{5 \times 10^{-9}}{\sqrt{2}} + \frac{10 \times 10^{-9}}{\sqrt{2}} + \frac{-5 \times 10^{-9}}{\sqrt{2}}\right) = \frac{9 \times 10^9 \times 10^{-9}}{\sqrt{2}}(5 + 10 - 5) = \frac{90}{\sqrt{2}}$$

$$V_O = 63.63\,V$$

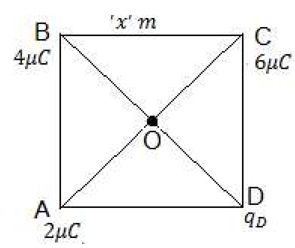
$$V_D = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AD} + \frac{q_B}{BD} + \frac{q_C}{CD}\right)$$

$$V_D = 9 \times 10^9\left(\frac{5 \times 10^{-9}}{2} + \frac{10 \times 10^{-9}}{2\sqrt{2}} + \frac{-5 \times 10^{-9}}{2}\right) = \frac{9 \times 10^9 \times 10^{-9}}{2}\left(5 + \frac{10}{\sqrt{2}} - 5\right) = \frac{90}{2\sqrt{2}}$$

$$V_D = 31.82\,V$$

$$W = (V_o - V_D) \times q = (63.63 - 31.82) \times 5 \times 10^{-6} = 31.81 \times 5 \times 10^{-6} = 159.05 \times 10^{-6}\,J$$

3) **Charges $2\mu C$, $4\mu C$ and $6\mu C$ are placed at three corners A, B and C respectively of a square $ABCD$ of side x metre. Find, what charge must be placed at the fourth corner so that the total potential at the centre of the square is zero. (July 2016)**



Given: $q_A = 2\,\mu C = 2 \times 10^{-6}\,C$
$q_B = 4\,\mu C = 4 \times 10^{-6}\,C$
$q_C = 6\,\mu C = 6 \times 10^{-6}\,C$
$Side, l = x\,m$

Potential at O will be zero when potential due to all the charges at O is zero.

Total potential at O is, $V_O = V_{AO} + V_{BO} + V_{CO} + V_{DO}$

If $V_O = 0, V_{AO} + V_{BO} + V_{CO} + V_{DO} = 0$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{AO} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{BO} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{CO} + \frac{1}{4\pi\varepsilon_0}\frac{q_D}{DO} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AO} + \frac{q_B}{BO} + \frac{q_C}{CO} + \frac{q_D}{DO}\right) = 0$$

But $AO = BO = CO = DO = x\sqrt{2}$

$$\frac{1}{4\pi\varepsilon_0} \times \frac{1}{x\sqrt{2}}(q_A + q_B + q_C + q_D) = 0$$

$q_A + q_B + q_C + q_D = 0 \qquad since\ \frac{1}{4\pi\varepsilon_0} \times \frac{1}{x\sqrt{2}} \neq 0$

$q_D = -(q_A + q_B + q_C) = -(2\mu C + 4\mu C + 6\mu C) = -12\ \mu C$

4) In parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} m^2$ and the distance between the plates is $3mm$. Calculate the capacitance of the capacitor. If this capacitor is connected to a $100V$ supply, what is the charge on each plate of the capacitor? (Absolute permittivity of free space=$8.85 \times 10^{-12} Fm^{-1}$) (Mar 2014)

Given: $A = 6 \times 10^{-3} m^2$

$d = 3\ mm = 3 \times 10^{-3}\ m$

$V = 100\ V$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F\ m^{-1}$

$C = \frac{\varepsilon_0 A}{d}$

$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 8.85 \times 2 \times 10^{-12} = 17.70 \times 10^{-12}\ F$

$C = \frac{Q}{V}$

$Q = C\ V = 17.70 \times 10^{-12} \times 100 = 17.70 \times 10^{-10}\ C = 1.77\ n\ C$

5) Three capacitors of capacitance $2pF$, $3pF$ and $4pF$ are connected in parallel.
(a) What is the total capacitance of the combination? (b) Determine the charge on each capacitor, if the combination is connected to a 100V supply.

Given: $C_1 = 2\ pF = 2 \times 10^{-12}\ F$

$C_2 = 3\ pF = 3 \times 10^{-12}\ F$

$C_3 = 4\ pF = 4 \times 10^{-12}\ F$

$V = 100\ V$

(a) For parallel combination, $C = C_1 + C_2 + C_3$

$C = 2 \times 10^{-12} + 3 \times 10^{-12} + 4 \times 10^{-12} = 9 \times 10^{-12}\ F = 9\ p\ F$

(b) $C = \frac{Q}{V}$

$Q = C\ V$ and for parallel combination $V_1 = V_2 = V_3 = V$

(i) $Q_1 = C_1 V = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10} = 0.2\ n\ C$

(ii) $Q_2 = C_2 V = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10} = 0.3\ n\ C$

(iii) $Q_3 = C_3 V = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10} = 0.4\ n\ C$

6) The plates of a parallel plate capacitor have an area of $100\ cm^2$ each and are separated by $3\ mm$. The capacitor is charged by connecting it to a $400\ V$ supply.
(a) Calculate the electrostatic energy stored in the capacitor.
(b) If a dielectric of dielectric constant 2.5 is introduced between the plates of the capacitor, then find the electrostatic energy stored and also change in the energy stored. (Jul 2018)

Given: $A = 100\ cm^2 = 100\ cm \times cm = 10^2 \times 10^{-2}\ m \times 10^{-2} m = 10^{-2} m^2$

$d = 3\ mm = 3 \times 10^{-3}\ m$

$V = 400\ V$

$K = 2.5$

(a)

$$E = \frac{1}{2} C V^2$$

Calculate C first.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{8.85 \times 10^{-12} \times 10^{-2}}{3 \times 10^{-3}} = \frac{8.85}{3} \times 10^{-11} = 2.95 \times 10^{-11}\ F$$

$$E = \frac{1}{2} \times 2.95 \times 10^{-11} \times 400 \times 400 = 236000 \times 10^{-11}\ J = 2.36 \times 10^{-6}\ J$$

(b) If a dielectric is introduced between the plates of a capacitor, the capacitance increases by K times, where K is called dielectric constant.

$$C' = CK$$

$$E' = \frac{1}{2} C' V^2 = \frac{1}{2} (CK) V^2 = K \times \frac{1}{2} C V^2 = KE$$

$$E' = 2.5 \times 2.36 \times 10^{-6} = 5.9 \times 10^{-6}\ J$$

Change in energy, $\Delta E = E' - E = 5.9 \times 10^{-6} - 2.36 \times 10^{-6} = 3.54 \times 10^{-6}\ J$

7) Two capacitors of $3\mu F$ and $5\mu F$ are connected in series. Calculate the equivalent capacitance. If a battery of emf 10V is connected across them, Calculate, (a) the charge on each capacitor and (b) the potential difference across each other.

Given: $C_1 = 3\ \mu F = 3 \times 10^{-6}\ F$

$C_2 = 5\ \mu F = 5 \times 10^{-6}\ F$

$V = 10\ V$

For series connection,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 10^{-6} \times 5 \times 10^{-6}}{3 \times 10^{-6} + 5 \times 10^{-6}} = \frac{15 \times 10^{-6} \times 10^{-6}}{8 \times 10^{-6}} = \frac{15}{8} \times 10^{-6} = 1.875 \times 10^{-6}$$

$$C = 1.875\ \mu\ F$$

(a) For series combination, $Q_1 = Q_2 = Q$

$$C = \frac{Q}{V}$$

$$Q = CV = 1.875 \times 10^{-6} \times 10 = 18.75 \times 10^{-6}\ C$$

(b)

$$V_1 = \frac{Q}{C_1} = \frac{18.75 \times 10^{-6}}{3 \times 10^{-6}} = 6.25\ V$$

$$V_2 = \frac{Q}{C_2} = \frac{18.75 \times 10^{-6}}{5 \times 10^{-6}} = 3.75\ V$$

8) Two capacitors of $30\mu F$ and $40\mu F$ are charged to $100V$ and $80V$ respectively. If they are connected in parallel, calculate the energy stored and loss of energy due to connection.

Given: $C_1 = 30\ \mu F = 30 \times 10^{-6}\ F$
$C_2 = 40\ \mu F = 40 \times 10^{-6}\ F$
$V_1 = 100\ V$
$V_2 = 80\ V$

$$E = \frac{1}{2}(C_1 + C_2)V^2$$

where V is called common potential difference, when they are connected in parallel.

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{30 \times 10^{-6} \times 100 + 40 \times 10^{-6} \times 80}{30 \times 10^{-6} + 40 \times 10^{-6}} = \frac{3000 \times 10^{-6} + 3200 \times 10^{-6}}{70 \times 10^{-6}}$$

$$V = \frac{6200 \times 10^{-6}}{70 \times 10^{-6}} = 88.57\ V$$

$$E = \frac{1}{2} \times (30 \times 10^{-6} + 40 \times 10^{-6}) \times (88.57)^{\ 2} = \frac{1}{2} \times 70 \times 10^{-6} \times 7844.64$$

$$E = 274562.57 \times 10^{-6} = 0.2745\ J$$

$$\Delta E = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 \sim V_2)^2$$

$$\Delta E = \frac{1}{2} \times \left(\frac{30 \times 10^{-6} + 40 \times 10^{-6}}{30 \times 10^{-6} + 40 \times 10^{-6}}\right) \times (80 \sim 100)^2 = \frac{1}{2} \times \frac{1200 \times 10^{-6} \times 10^{-6}}{70 \times 10^{-6}} \times 20^2$$

$$\Delta E = 3400 \times 10^{-6} = 0.0034\ J$$

9) **The effective capacitance of two capacitors is $7\ \mu F$ when in parallel and $\frac{6}{7}\ \mu F$ when in series. Find the individual capacitance.**

Given: $C_P = 7\ \mu F = 7 \times 10^{-6}\ F$
$C_S = \frac{6}{7}\ \mu F = \frac{6}{7} \times 10^{-6}\ F$

For Parallel combination, $C_P = C_1 + C_2$

$C_1 + C_2 = 7 \times 10^{-6} \quad ----(1)$

For Series combination,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \Longrightarrow C_S = \frac{C_1C_2}{C_1 + C_2}$$

$$C_1C_2 = (C_1 + C_2)C_S = 7 \times 10^{-6} \times \frac{6}{7} \times 10^{-6}$$

$C_1C_2 = 6 \times 10^{-12} \quad ----(2)$

We have, $(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4\ C_1C_2$

$(C_1 - C_2)^2 = (7 \times 10^{-6})^2 - 4 \times 6 \times 10^{-12} = 49 \times 10^{-12} - 24 \times 10^{-12}$

$(C_1 - C_2)^2 = 25 \times 10^{-12}$

$C_1 - C_2 = 5 \times 10^{-6} \quad ----(3)$

Solving equations (1) and (3)

$C_1 + C_2 = 7 \times 10^{-6}$

$C_1 - C_2 = 5 \times 10^{-6}$

$2C_1 = 12 \times 10^{-6}$

$C_1 = 6 \times 10^{-6}$

Substituting for C_1 in equation (1), $C_1 + C_2 = 7 \times 10^{-6}$

$C_2 = 7 \times 10^{-6} - C_1 = 7 \times 10^{-6} - 6 \times 10^{-6} = 1 \times 10^{-6}$

$C_1 = 6 \times 10^{-6}\ F$ and $C_2 = 7 \times 10^{-6}\ F$

10) A capacitor of capacitance $5\mu F$ is charged to potential of $500V$. Then it is disconnected from the battery and connected to uncharged capacitor of capacitance $3\mu F$. Calculate the common potential, charge on each capacitor and the loss of energy.

Given: $C_1 = 5\ \mu F = 5 \times 10^{-6}\ F$

$C_2 = 3\ \mu F = 3 \times 10^{-6}\ F$

$V_1 = 500\ V$

$V_2 = 0\ V$

When two capacitors (one charged and another uncharged) are connected, there is redistribution of charges and each capacitor acquires same potential called common potential (V).

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{5 \times 10^{-6} \times 500 + 3 \times 10^{-6} \times 0}{5 \times 10^{-6} + 3 \times 10^{-6}} = \frac{2500 \times 10^{-6}}{8 \times 10^{-6}} = 312.5\ V$$

$Q_1 = C_1V = 5 \times 10^{-6} \times 312.5 = 1562.5 \times 10^{-6}\ F$

$Q_2 = C_2V = 3 \times 10^{-6} \times 312.5 = 937.5 \times 10^{-6}\ F$

$$\Delta E = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 \sim V_2)^2$$

$$\Delta E = \frac{1}{2} \times \left(\frac{5 \times 10^{-6} + 3 \times 10^{-6}}{5 \times 10^{-6} + 3 \times 10^{-6}}\right) \times (500 \sim 0)^2 = \frac{1}{2} \times \frac{15 \times 10^{-6} \times 10^{-6}}{8 \times 10^{-6}} \times 500^2$$

$$\Delta E = \frac{1}{2} \times \frac{15 \times 10^{-6}}{8} \times 250000 = 23.43 \times 10^{-2} = 0.2343\ J$$

11) When two capacitors are connected in series and connected across 4kV line, the energy stored in the system is 8 J. The same capacitors, if connected in parallel across the same line, the energy stored is 36 J. Find the individual capacitances. (Mar 2016)

Given: $V = 4\ kV = 4 \times 10^3\ V$

$U_S = 8\ J$

$U_P = 36\ J$

For parallel combination,

$$U_P = \frac{1}{2}\ C_P\ V^2$$

$$U_P = \frac{1}{2}\ (C_1 + C_2)\ V^2$$

$$36 = \frac{1}{2} \times (C_1 + C_2) \times (4 \times 10^3)^2$$

$$C_1 + C_2 = \frac{36 \times 2}{(4 \times 10^3)^2} = \frac{36 \times 2}{16 \times 10^6} = 4.5 \times 10^{-6}\ \ ----(1)$$

For series combination,

$$U_S = \frac{1}{2}\ C_S\ V^2$$

$$U_P = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right) V^2$$

$$8 = \frac{1}{2} \times \left(\frac{C_1 C_2}{C_1 + C_2}\right) \times (4 \times 10^3)^2$$

$$\frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 2}{(4 \times 10^3)^2} = \frac{16}{16 \times 10^6} = 10^{-6}$$

$$C_1 C_2 = (C_1 + C_2) \times 10^{-6} = 4.5 \times 10^{-6} \times 10^{-6} = 4.5 \times 10^{-12} \quad ----(2)$$

We have, $(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4\, C_1 C_2$

$$(C_1 - C_2)^2 = (4.5 \times 10^{-6})^2 - 4 \times 4.5 \times 10^{-12} = 20.25 \times 10^{-12} - 18 \times 10^{-12}$$

$$(C_1 - C_2)^2 = 2.25 \times 10^{-12}$$

$$C_1 - C_2 = 1.5 \times 10^{-6} \quad ----(3)$$

Solving equations (1) and (3)

$$C_1 + C_2 = 4.5 \times 10^{-6}$$

$$C_1 - C_2 = 1.5 \times 10^{-6}$$

$$2C_1 = 6.0 \times 10^{-6}$$

$$C_1 = 3 \times 10^{-6}$$

Substituting for C_1 in equation (1), $C_1 + C_2 = 4.5 \times 10^{-6}$

$$C_2 = 4.5 \times 10^{-6} - C_1 = 4.5 \times 10^{-6} - 3 \times 10^{-6} = 1.5 \times 10^{-6}$$

$C_1 = 3 \times 10^{-6}\, F$ and $C_2 = 1.5 \times 10^{-6}\, F$

Numerical Problems:

1) **Calculate the current density and average drift speed of conduction electrons in a copper wire of cross sectional area $1.0 \times 10^{-7}\, m^2$ carrying a current of $1.5\, A$. Given free electron density of copper is $8 \times 10^{28}\, electrons/m^3$, $e = 1.6 \times 10^{-19}\, C$.**

Given: $A = 1.0 \times 10^{-7}\, m^2$

$I = 1.5\, A$

$n = 8 \times 10^{28}\, electrons/m^3$

$e = 1.6 \times 10^{-19}\, C$

$$j = \frac{I}{A} = \frac{1.5}{1.0 \times 10^{-7}} = 1.5 \times 10^7\, Am^2$$

$$I = neAv_d$$

$$v_d = \frac{I}{neA} = \frac{1.5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}} = \frac{1.5}{8 \times 1.6 \times 10^2} = 0.117 \times 10^{-2}$$

$$v_d = 1.17 \times 10^{-3}\, m\, s^{-1}$$

2) **The number density of free electrons in a copper conductor is $8.5 \times 10^{28}\, \mathrm{m}^{-3}$. How long does an electron take to drift from one end of a wire $3.0\, \mathrm{m}$ long to its other end? The area of cross section of the wire is $2.0 \times 10^{-6}\, \mathrm{m}^2$ and it is carrying a current of $3.0\mathrm{A}$.**

Given: $n = 8.5 \times 10^{28}\, m^{-3}$

$l = 3\, m$

$A = 2.0 \times 10^{-6}\, m^2$

$I = 3\, A$

$$I = neAv_d$$

$$v_d = \frac{I}{neA} = \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-7}} = \frac{3}{27.2 \times 10^3} = 0.11029 \times 10^{-3}\, m\, s^{-1}$$

Time taken to drift 3 m long can be calculated as,

$$velocity = \frac{Distance\ travelled}{time\ taken}$$

$$v_d = \frac{l}{t}$$

$$t = \frac{l}{v_d} = \frac{3}{0.11029 \times 10^{-3}} = 27.20 \times 10^3\ second$$

3) **A silver wire has a resistance of $2.1\, \Omega$ at $27.5^0 C$ and a resistance of $2.7\, \Omega$ at $100^0\, C$. Find the temperature co-efficient of silver. Also find its resistance at $0^0 C$.**

Given: $R_0 = 2.1\, \Omega$ $T_0 = 27.5^0$

$R = 2.7\, \Omega$ $T = 100^0$

$$\alpha = \frac{R - R_0}{R_0(T - T_0)}$$

$$\alpha = \frac{2.7 - 2.1}{2.1(100 - 27.5)} = \frac{0.6}{2.1 \times 72.5} = 0.00394088\ {}^0C^{-1}$$

If we take $R = 2.1\,\Omega$ and $T = 27.5^0$, the resistance at zero degree ($T_0 = 0^0$) is taken as R_0

$$R = R_0[1 + \alpha(T - T_0)]$$

$$R_0 = \frac{R}{1 + \alpha(T - T_0)} = \frac{2.1}{1 + 0.00394(27.5 - 0)} = \frac{2.1}{1 + 0.1084} = \frac{2.1}{1.1084} = 1.89\,\Omega$$

4) **$100\,mg$ mass of nichrome metal is drawn in to a wire of area of cross-section $0.05\,mm^2$. Calculate the resistance of the wire. Given density of nichrome $8.4 \times 10^3\,kg\,m^{-3}$ and resistivity of the material as $1.2 \times 10^{-6}\,\Omega\,m$. (Mar 2018)**

Given: $m = 100\,mg = 100 \times 10^{-3} \times 10^{-3}\,kg = 100 \times 10^{-6}\,kg$

$A = 0.05\,mm^2 = 0.05\,mm \times mm = 0.05 \times 10^{-3}\,m \times 10^{-3}\,m = 0.05 \times 10^{-6}\,m^2$

$Density, D = 8.4 \times 10^3\,kg\,m^{-3}$

$\rho = 1.2 \times 10^{-6}\,\Omega\,m$

$$D = \frac{mass}{volume} = \frac{m}{Al}$$

$$l = \frac{m}{D \times A}$$

$$Resistance, \quad R = \rho\frac{l}{A}$$

$$R = \rho\left(\frac{m}{D \times A}\right)\frac{1}{A} = 1.2 \times 10^{-6} \times \frac{100 \times 10^{-6}}{8.4 \times 10^3 \times 0.05 \times 10^{-6}} \times \frac{1}{0.05 \times 10^{-6}}$$

$$R = \frac{1.2 \times 10^{-1}}{8.4 \times 0.05 \times 0.05} = 5.72\,\Omega$$

5) **A wire having length $2.0\,m$, diameter $1.0\,mm$ and resistivity $1.963 \times 10^{-8}\,\Omega\,m$ is connected in series with a battery of emf $3\,V$ and internal resistance $1\,\Omega$.. Calculate the resistance of the wire and current in the circuit. (July 2016)**

Given: $l = 2.0\,m$

$Diameter,\ d = 1\,mm = 1 \times 10^{-3}\,m$

$\rho = 1.963 \times 10^{-8}\,\Omega\,m$

$\varepsilon = 3\,V,\ r = 1\,\Omega$

$$R = \frac{\rho l}{A}$$

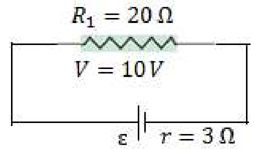
First calculate A, $A = \pi(radius)^2$

$$A = \pi\left(\frac{d}{2}\right)^2 = 3.14 \times \left(\frac{1 \times 10^{-3}}{2}\right)^2 = \frac{3.14 \times 10^{-6}}{4} = 0.785 \times 10^{-6}\,m^2$$

$$R = \frac{1.963 \times 10^{-8} \times 2.0}{0.785 \times 10^{-6}} = 5.0012 \times 10^{-2} = 0.05\,\Omega$$

$$I = \frac{\varepsilon}{R + r} = \frac{3}{0.05 + 1} = \frac{3}{1.05} = 2.85\,A$$

6) A battery of internal resistance 3 Ω is connected to 20 Ω resistor and potential difference across the resistor is 10 V. If another resistor of 30Ω is connected in series with the first resistor and the battery again connected to the combination. Calculate the emf and the terminal potential difference across the combination. (Mar 2014)



Case (i)

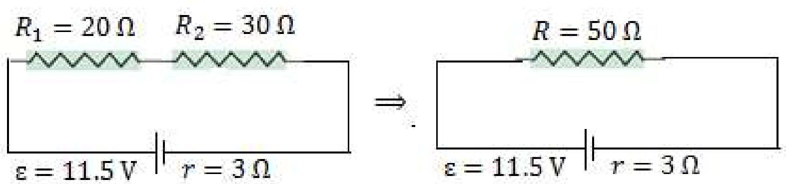
Given: $R_1 = 20\,\Omega,\ V = 100\,V,\ r = 3\,\Omega$

First calculate emf of the battery using the expression for V

$$V = \left(\frac{\varepsilon}{R_1 + r}\right) R_1$$

$$\varepsilon = \frac{V(R_1 + r)}{R_1} = \frac{10 \times (20 + 3)}{20} = \frac{10 \times 23}{20} = 11.5\,V$$

Case(ii)



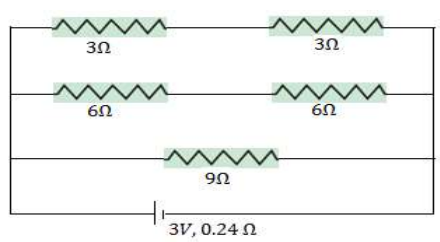
$$R = R_1 + R_2 = 20 + 30 = 50\,\Omega$$

Terminal potential difference, V is given by

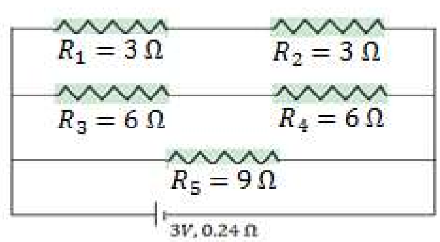
$$V = \left(\frac{\varepsilon}{R + r}\right) R$$

$$V = \left(\frac{11.5}{50 + 3}\right) \times 50 = \frac{575}{53} = 10.85\,V$$

7) In the given circuit diagram, calculate:
(i) The main current through the circuit and
(ii) Also current through 9 Ω resistor. (Jul 2018)



Given: $R_1 = 3\,\Omega,\ R_2 = 3\,\Omega, R_3 = 6\,\Omega$
$R_4 = 6\,\Omega, R_5 = 9\,\Omega$



$R_1\ series\ R_2 = R_1 + R_2 = 3 + 3 = 6\,\Omega$

$R_3\ series\ R_4 = R_3 + R_4 = 6 + 6 = 12\,\Omega$

Total resistance of the circuit is given by

$$\frac{1}{R} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} + \frac{1}{R_5}$$

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{12} + \frac{1}{9} = \frac{6 + 3 + 4}{36} = \frac{13}{36}$$

$$R = \frac{26}{36} = 2.76\,\Omega$$

$$I = \frac{\varepsilon}{R + r} = \frac{3}{2.76 + 0.24} = \frac{3}{3} = 1\,A$$

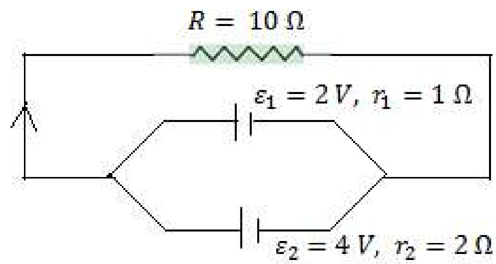
$$V = \left(\frac{\varepsilon}{R + r}\right) R$$

$$V = \left(\frac{3}{2.76 + 0.24}\right) \times 2.76 = 2.76\,V$$

Current through 9 Ω resistor can be calculated as,

$$I = \frac{V}{R_5} = \frac{2.76}{9} = 0.367\,A$$

8) **Two cells of emf 2 V and 4 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send the current in the same direction through an external resistance of 10 Ω. Find the potential difference across 10 Ω resistor. (Mar 2015)**



Given: $R = 10\,\Omega$

$\varepsilon_1 = 2\,V,\ r_1 = 1\,\Omega$

$\varepsilon_2 = 4\,V,\ r_2 = 2\,\Omega$

First calculate effective emf and effective internal resistance of the equivalent battery.
For parallel combination of cells,

$$\varepsilon_{eff} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$\varepsilon_{eff} = \frac{2 \times 2 + 4 \times 1}{1 + 2} = \frac{8}{3} = 2.67\,V$$

$$r_{eff} = \frac{r_1 r_2}{r_1 + r_2} = \frac{1 \times 2}{1 + 2} = \frac{2}{3} = 0.67\,\Omega$$

Potential difference can be calculated as

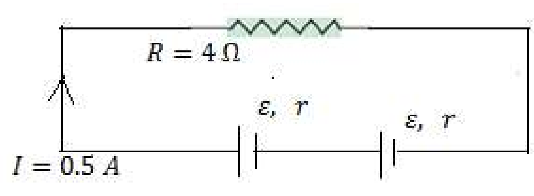
$$V = \left(\frac{\varepsilon_{eff}}{R + r_{eff}}\right) R$$

$$V = \left(\frac{2.67}{10 + 0.67}\right) \times 10 = \frac{2.67 \times 10}{10.67} = 2.5\,V$$

9) **Two identical cells either in series or in parallel combination, gives the same current of 0.5 A through external resistance of 4 Ω. Find emf and internal resistance of each cell. (Jul 2015)**

Given: $I = 0.5\,A$

$R = 4\,\Omega$



Cells are identical $\Longrightarrow \varepsilon_1 = \varepsilon_2 = \varepsilon$ and $r_1 = r_2 = r$

Case(i)
For series combination,

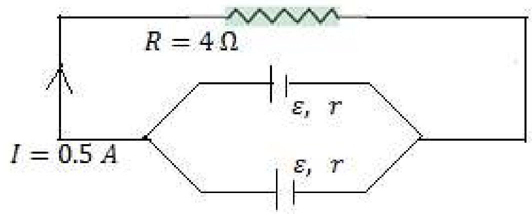
$\varepsilon_{eff} = \varepsilon + \varepsilon = 2\varepsilon$ and $r_{eff} = r + r = 2r$

$$I = \frac{\varepsilon_{eff}}{R + r_{eff}}$$

$$0.5 = \frac{2\varepsilon}{4 + 2r}$$

$$2\varepsilon = 2 + r$$

$2\varepsilon - r = 2 \ ---(1)$



Case(ii)

For parallel combination,

$$\varepsilon_{eff} = \frac{\varepsilon r + \varepsilon r}{r + r} = \frac{2\varepsilon r}{2r} = \varepsilon$$

$$r_{eff} = \frac{rr}{r + r} = \frac{r^2}{2r} = \frac{r}{2}$$

$$I = \frac{\varepsilon_{eff}}{R + r_{eff}}$$

$$0.5 = \frac{\varepsilon}{4 + \frac{r}{2}}$$

$$\varepsilon = 2 + \frac{r}{4} \Longrightarrow 4\varepsilon = 8 + r$$

$$4\varepsilon - r = 8 \ ---(2)$$

Solving equation (1) and equation (2) we get,

$\varepsilon = 3\,V$ and $r = 4\,\Omega$

10) A cell supplies $0.9\,A$ current through $2\,\Omega$ resistor and a current of $0.3\,A$ through $7\,\Omega$ resistor. Find the internal resistance and emf of the cell.

Given: $I_1 = 0.9\,A$ and $R_1 = 2\,\Omega$

$I_2 = 0.3\,A$ and $R_2 = 7\,\Omega$

Case(i) $I_1 = 0.9\,A$ and $R_1 = 2\,\Omega$

$$I_1 = \frac{\varepsilon}{R_1 + r}$$

$$0.9 = \frac{\varepsilon}{2 + r}$$

$$\varepsilon = 1.8 + 0.9\,r \ ----(1)$$

Case(ii) $I_2 = 0.3\,A$ and $R_2 = 7\,\Omega$

$$I_2 = \frac{\varepsilon}{R_2 + r}$$

$$0.3 = \frac{\varepsilon}{7 + r}$$

$$\varepsilon = 2.1 + 0.3\,r \ ----(2)$$

From (1) and (2) $1.8 + 0.9\,r = 2.1 + 0.3\,r$

$0.9\,r - 0.3\,r = 2.1 - 1.8$

$0.6\,r = 0.3$

$r = 0.5\,\Omega$

Substituting the value of r in equation (1)

$\varepsilon = 1.8 + 0.9 \times 0.5 = 1.8 + 0.45 = 2.25\,V$

11) When two resistors are connected in series with a cell of emf $2\,V$ and negligible internal resistance, a current of $\frac{2}{5}\,A$ flows in the circuit. When the resistors are connected in parallel the main current is $\frac{5}{3}\,A$. Calculate the resistances. (Mar 2017)

Given: $\varepsilon = 2\,V$

$I_1 = \frac{2}{5}\,A$

$I_2 = \frac{5}{3}\,A$

$$I = \frac{\varepsilon}{R + r}$$

$$R = \frac{\varepsilon}{I} \qquad \text{(Since,} \quad r = 0)$$

For Series connection, $R_S = R_1 + R_2$

$$R_S = \frac{\varepsilon}{I_1}$$

$$R_1 + R_2 = \frac{2}{\left(2/5\right)}$$

$$R_1 + R_2 = 5\,\Omega \quad ----(\text{i})$$

For Parallel Connection, $R_P = \frac{R_1 R_2}{R_1 + R_2}$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{\varepsilon}{I_2} = \frac{2}{\left(5/3\right)} = \frac{6}{5}\,\Omega$$

$$R_1 R_2 = (R_1 + R_2) \times \frac{6}{5} = 5 \times \frac{6}{5}$$

$$R_1 R_2 = 6\,\Omega \quad ----(\text{ii})$$

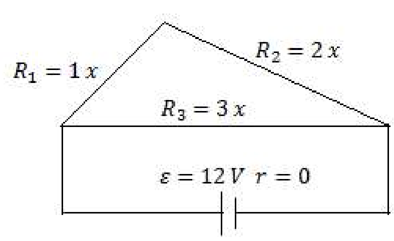
$$(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4R_1R_2$$

$$(R_1 - R_2)^2 = 25 - 24$$

$$R_1 - R_2 = 1\,\Omega \quad ----(\text{iii})$$

Solving Equation (i) and (iii), we get, $R_1 = 3\,\Omega$ and $R_2 = 1\,\Omega$

12) A uniform wire of resistance $12\,\Omega$ is bent into three parts in the ratio $1:2:3$ to form a triangle. A cell of emf $12\,V$ and negligible internal resistance is connected across the highest of the three resistances. Calculate the current in each branch of the circuit.



Given: $R = 12\,\Omega$

$1\,x + 2\,x + 3\,x = 12$

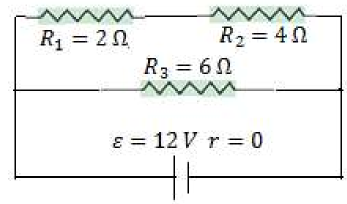
$x = 2\,\Omega$

$R_1 = 1 \times 2 = 2\,\Omega,\;\; R_2 = 2 \times 2 = 4\,\Omega,\;\; R_3 = 3 \times 2 = 6\,\Omega$

Total resistance of the circuit can be calculated as,

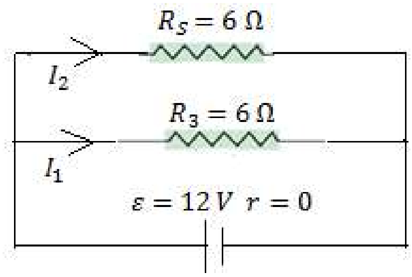
$$R_S = R_1 + R_2 = 2 + 4 = 6\ \Omega$$

$$R_P = \frac{R_S R_3}{R_S + R_3} = \frac{6 \times 6}{6 + 6} = 3\ \Omega$$



Total current I can be calculated as,

$$I = \frac{\varepsilon}{R_P + r} = \frac{12}{3 + 0} = 4\ A$$

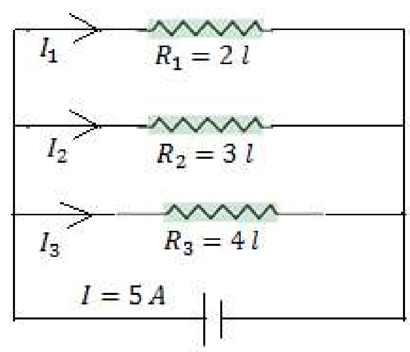


Current through R_3 and R_S is same, since each branch has same resistance.

$$I_1 = \frac{I}{2} = \frac{4}{2} = 2\ A$$

$$I_2 = \frac{I}{2} = \frac{4}{2} = 2\ A$$

13) An electric current 5 A divides into 3 parallel branches in which the lengths of the wires are in the ratio $2:3:4$ and the diameters are in the ratio $3:4:5$. Find the current in each branch, if the wires are of the same material.



Given: Total current, $I = 5\ A$

Ratio of length $= l_1 : l_2 : l_3 = 2:3:4$

Ratio of their diameter $= d_1 : d_2 : d_3 = 3:4:5$

The ratio of their radius will be $= r_1 : r_2 : r_3 = 3:4:5$

The ratio of their Area $= A_1 : A_2 : A_3 = 9:16:25$

(since area $A = \pi r^2$)

We have,

$$R = \rho \frac{l}{A}$$

$$R_1 : R_2 : R_3 = \frac{l_1}{A_1} : \frac{l_2}{A_2} : \frac{l_3}{A_3} = \frac{2}{9} : \frac{3}{16} : \frac{4}{25}$$

In parallel combination

$$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} = \frac{9}{2} : \frac{16}{3} : \frac{25}{4}$$

$$I_1 : I_2 : I_3 = \frac{9}{2} \times 12 : \frac{16}{3} \times 12 : \frac{25}{4} \times 12 \qquad (LCM\ of\ 2, 3\ and\ 4\ is\ 12)$$

$$I_1 : I_2 : I_3 = 54 : 64 : 75$$

But, $I_1 + I_2 + I_3 = I$

$54\,x + 64\,x + 75\,x = 5$

$193\,x = 5$

$x = 0.0259$

$I_1 = 54\,x = 54 \times 0.0259 = 1.398\ A$

$I_2 = 64\,x = 64 \times 0.0259 = 1.657\ A$

$I_3 = 75\,x = 75 \times 0.0259 = 1.942\ A$

Numerical Problems:

1) **A $20\,\Omega$ resistor, $1.5\,H$ inductor and $35\,\mu F$ capacitor are connected in series with a $220\,V, 50\,Hz$ ac supply. Calculate the impedance of the circuit and also find the current through the circuit. (Jul 2018)**

Given: $R = 20\,\Omega$
$L = 1.5\,H$
$C = 35\,\mu F = 35 \times 10^{-6}\,F$
$V(V_{rms}) = 220\,V,\ \nu = 50\,Hz$

(i) Find X_C and X_L then Z.
(ii) Find $I(I_{rms})$
In this Chapter current means RMS value of current and voltage means RMS value of voltage. Frequency means frequency of applied AC voltage.

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 35 \times 10^{-6}} = \frac{10^6}{10990} = 10^6 \times 90.99 \times 10^{-6} = 90.99\,\Omega$$
$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 1.5 = 471\,\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{20^2 + (90.99 - 471)^2} = \sqrt{20^2 + (-380)^2} = \sqrt{400 + 144400}$$
$$Z = \sqrt{144800} = 380.53\,\Omega$$

$$I(I_{rms}) = \frac{V_{rms}}{Z} = \frac{220}{380.53} = 0.578\,A$$

2) **A source of alternating emf of $220\,V - 50\,Hz$ is connected in series with a resistance of $220\,\Omega$ an inductance of $100\,mH$ and a capacitance of $30\,\mu F$. Does the current lead or lag the voltage and by what angle? (Mar 2017)**

Given: $R = 220\,\Omega$
$L = 100\,m\,H = 100 \times 10^{-3}\,H$
$C = 30\,\mu F = 30 \times 10^{-6}\,F$
$V(V_{rms}) = 220\,V,\ \nu = 50\,Hz$

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 30 \times 10^{-6}} = \frac{10^6}{9420} = 10^6 \times 106.157 \times 10^{-6} = 106.157\,\Omega$$
$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 100 \times 10^{-3} = 31.4\,\Omega$$

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{106.157 - 31.4}{220} = \frac{74.76}{220} = 0.3398$$
$$\phi = \tan^{-1}(0.3398) = 18.76^0$$
ϕ is positive indicates that current leads.

3) **A sinusoidal voltage of peak value $283V$ and frequency $50Hz$ is applied to a series LCR circuit in which $R = 3\Omega$, $L = 25.48mH$ and $C = 786\mu F$. (Mar 2015, July 2019)**

Find: **a) Impedance of the circuit.**
b) The phase difference between the voltage across the source and the current.
c) The power factor.

Given: $R = 3\,\Omega$

$L = 25.48\,mH = 25.48 \times 10^{-3}\,H$

$C = 786\,\mu F = 786 \times 10^{-6}\,F$

$v_m = 283\,V,\ \nu = 50\,Hz$

(a)

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 786 \times 10^{-6}} = \frac{10^6}{246804} = 10^6 \times 4.0517 \times 10^{-6} = 4.0517\ \approx 4\Omega$$

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8000.72 \times 10^{-3} \approx 8\,\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{3^2 + (4-8)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16}$$

$$Z = \sqrt{25} = 5\,\Omega$$

(b)

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{4-8}{3} = \frac{-4}{3} = -1.3333$$

$$\phi = -\tan^{-1}(1.3333) = -53.1294^0$$

Negative sign indicates that voltage leads.

(c)

$$\cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6 \qquad \text{or} \qquad \cos\phi = \cos 53.1294^0 = 0.6$$

4) **Calculate the resonant frequency and Q-factor of a series LCR circuit containing a pure inductor of inductance 3H, capacitor of capacitance $27\mu F$ and resistor of resistance 7.4Ω.(Mar 2014)**

Given: $R = 7.4\,\Omega$

$L = 3\,H$

$C = 27\,\mu F = 27 \times 10^{-6}\,F$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{1}{9 \times 10^{-3}} = \frac{10^3}{9} = 0.1111 \times 10^3 = 111\,rad\,s^{-1}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (use\ any\ one\ formula)$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{7.4}\sqrt{\frac{3}{27 \times 10^{-6}}} = \frac{1}{7.4} \times \sqrt{\frac{1}{9 \times 10^{-6}}} = \frac{1}{7.4} \times \frac{1}{3 \times 10^{-3}} = \frac{10^3}{22.2} = 10^3 \times 0.045 = 45$$

5) **A sinusoidal voltage of peak value 285V is applied to a series LCR circuit in which resistor of resistance 5Ω, pure inductor of inductance 28.5 mH and capacitance of capacitor 800 μF are connected. a) Find resonant frequency. (Jul 2017)**
b) Calculate the impedance, current and power dissipated at the resistance.

Given: $R = 5\,\Omega$

$L = 28.5\,mH = 28.5 \times 10^{-3}\,H$

$C = 800\,\mu F = 800 \times 10^{-6}\,F$

$v_m = 285\,V$

(a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{28.5 \times 10^{-3} \times 800 \times 10^{-6}}} = \frac{1}{\sqrt{222800 \times 10^{-9}}} = \frac{1}{\sqrt{22280 \times 10^{-8}}} = 209.4\, rad\ s^{-1}$$

(b)

At resonance, $Z = R = 5\,\Omega$

$$V_{rms} = \frac{v_m}{\sqrt{2}} = \frac{285}{1.4142} = 201.5273\, V$$
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{201.5273}{5} = 40.30\, A$$

$$P = I_{rms}^2 \times R = 40.30^2 \times 5 = 8120.45 = 8.12\, kW$$

6) A pure inductor of 25mH is connected to a source of 220V and 50Hz. Find the inductance resistance, rms value of current and peak current in the circuit. (Jul 2014)

Given: $L = 25\, m\, H = 25 \times 10^{-3}\, H$

$V_{rms} = 220\, V,\ \ \nu = 50\, Hz$

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7850 \times 10^{-3} = 7.85\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{7.85} = 28\, A$$

$$i_m = \sqrt{2}\, I_{rms} = 1.4142 \times 28 = 39.59\, A$$

7) A resistor of 100Ω, a pure inductance coil of L = 0.5H and capacitor are in series in a circuit containing an AC source of 220V, 50Hz. In the circuit, current is ahead of the voltage by 30⁰. Find the value of the capacitor. (Jul 2015)

Given: $R = 100\,\Omega$

$L = 0.5\, H$

$V_{rms} = 220\, V,\ \ \nu = 50\, Hz$

$\phi = 30^0$

$$\tan\phi = \frac{X_C - X_L}{R}$$
$$X_C = R\tan\phi + X_L$$
$$\frac{1}{2\pi\nu C} = R\tan\phi + X_L$$
$$C = \frac{1}{2\pi\nu(R\tan\phi + 2\pi\nu L)}$$
$$C = \frac{1}{2 \times 3.14 \times 50(100 \times \tan 30^0 + 2 \times 3.14 \times 50 \times 0.5)}$$
$$C = \frac{1}{314 \times (100 \times 0.5774 + 157)} = \frac{1}{67828.36} = 14.74 \times 10^{-6}\, F$$

8) **An inductor and a bulb are connected in series to an AC source of $220\,V, 50Hz$. A current of $11\,A$ flows in the circuit and phase angle between the voltage and current is $\frac{\pi}{4}$ radians. Calculate the impedance and inductance of the circuit. (July 2016)**

Given: $V_{rms} = 220\,V,\ \nu = 50\,Hz$

$I_{rms} = 11\,A$

$\phi = \frac{\pi}{4}\ rad$

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{220}{11} = 20\,\Omega$$

$$X_L = 2\pi\nu \Longrightarrow L = \frac{X_L}{2\pi\nu}$$

We have,

$$\tan\phi = \frac{X_L}{R} \Longrightarrow X_L = R \times \tan\phi \quad \text{and} \quad \cos\phi = \frac{R}{Z} \Longrightarrow R = Z \times \cos\phi$$

$$L = \frac{R \times \tan\phi}{2\pi\nu} = \frac{Z \times \cos\phi \times \tan\phi}{2\pi\nu} = \frac{Z \times \sin\phi}{2\pi\nu} = \frac{20 \times \sin\left(\frac{\pi}{4}\right)}{2 \times 3.14 \times 50}$$

$$L = \frac{20 \times 0.7071}{314} = \frac{14.1421}{314} = 0.0450\,H$$

or

$$\tan\phi = \frac{X_L}{R}$$

$$\tan\frac{\pi}{4} = \frac{X_L}{R} \Longrightarrow 1 = \frac{X_L}{R}$$

$$X_L = R$$

Now, $Z = \sqrt{R^2 + (X_L)^2} = \sqrt{(X_L)^2 + (X_L)^2} = \sqrt{2(X_L)^2} = \sqrt{2} \times X_L$

$$X_L = \frac{Z}{\sqrt{2}}$$

$$2\pi\nu L = \frac{Z}{\sqrt{2}}$$

$$L = \frac{Z}{\sqrt{2} \times 2\pi\nu} = \frac{20}{1.4142 \times 2 \times 3.14 \times 50} = \frac{20}{444.0630} = 0.0450\,H$$

9) **A 60V, 10W lamp is to be run on 100V, 60 Hz ac mains. Calculate the inductance of a choke coil required to be connected in series with it to work the bulb.**

For bulb: $V_{rms} = 60\,V$

$P = 10\,W$

(i) $P = VI$

$$I = \frac{P}{V} = \frac{10}{60} = 0.1667\,A$$

(ii)

$$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{60^2}{10} = 360\ \Omega$$

For circuit: $R = 360\ \Omega$

$V_{rms} = 100\ V,\ \nu = 60\ Hz$

For proper functioning of bulb, current through the bulb must 0.1667 A and must not exceed 0.1667 A.

$$Z = \frac{V}{I}$$

$$\sqrt{R^2 + (X_L)^2} = \frac{V}{I}$$

$$R^2 + (X_L)^2 = \frac{V^2}{I^2}$$

$$X_L^2 = \frac{V^2}{I^2} - R^2$$

$$X_L = \sqrt{\frac{V^2}{I^2} - R^2} = \sqrt{\frac{100^2}{(0.1667)^2} - 360^2} = \sqrt{359856 - 129600} = \sqrt{230256} = 479.85$$

$$L = \frac{479.85}{2\pi\nu} = \frac{479.85}{2 \times 3.14 \times 60} = \frac{479.85}{376.8} = 1.273\ H$$

10) A series L C R circuit with $R = 20\ \Omega, L = 1.5\ H$ and $C = 35\mu F$ is connected to a variable frequency of 200 V AC supply when the frequency of the supply is equal to the natural frequency of the circuit, What is the average power transferred to the circuit in one complete cycle.

Given: $R = 20\ \Omega$

$L = 1.5\ H$

$C = 35\ \mu F = 35 \times 10^{-6}\ F$

$V(V_{rms}) = 200\ V$

When the frequency of the supply is equal to the natural frequency of the circuit, current will be maximum.

i.e. $X_C = X_L$ and $Z = R$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{20} = 10\ A$$

$$P = V_{rms} I_{rms} = 200 \times 10 = 2000\ W$$

or

$$P = \frac{V_{rms}^2}{R} = \frac{200^2}{20} = 2000\ W$$

11) A $\left(\frac{1}{12\pi}\right) mF$ capacitor in series with a 40 Ω resistor is connected to a $110V$, $60Hz$ supply.

(a) What is the maximum current in the circuit?

(b) What is the phase difference between the current maximum and voltage maximum?

Given: $C = \frac{1}{12\pi}\ mF = \frac{1}{12\pi} \times 10^{-3}\ F$

$R = 40\ \Omega$

$V_{rms} = 110\ V,\ \nu = 60\ Hz$

(a)

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2 \times \pi \times 60 \times \frac{1}{12\pi} \times 10^{-3}} = \frac{1}{10 \times 10^{-3}} = \frac{10^3}{10} = 100\ \Omega$$

$$Z = \sqrt{R^2 + (X_C)^2} = \sqrt{40^2 + 100^2} = \sqrt{1600 + 10000} = \sqrt{11600} = 107.7\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{110}{107.7} = 1.02\ A$$

$$i_m = \sqrt{2}\, I_{rms} = 1.4142 \times 1.02 = 1.442\ A$$

(b)

$$\tan\phi = \frac{X_C}{R} = \frac{100}{40} = 2.5$$

$$\phi = \tan^{-1}(2.5) = 68.2^0$$

12) A coil of inductance $\mathbf{0.50}H$ and resistance of $\mathbf{100\ \Omega}$ is connected to a $\mathbf{240}V, \mathbf{50}Hz$ ac supply.
a) What is the maximum current in the coil?
b) What is the time lag between voltage maximum and the current maximum?

Given: $L = 0.50\ H$
$R = 100\ \Omega$
$V_{rms} = 240\ V,\ \nu = 50\ Hz$

(a)

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 0.50 = 157\ \Omega$$

$$Z = \sqrt{R^2 + (X_L)^2} = \sqrt{100^2 + 157^2} = \sqrt{10000 + 24649} = \sqrt{34649} = 186.14\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{240}{186.14} = 1.29\ A$$

$$i_m = \sqrt{2}\, I_{rms} = 1.4142 \times 1.29 = 1.824\ A$$

(b)

$$\tan\phi = \frac{X_L}{R} = \frac{157}{100} = 1.57$$

$$\phi = \tan^{-1}(1.57) = 57.5^0$$

Convert the value of ϕ in to radian.

$$\phi = 57.5 \times \frac{\pi}{180} = 1.003\ rad$$

We have, $\omega = \frac{\phi}{t}$

$$t = \frac{\phi}{\omega} = \frac{\phi}{2\pi\nu} = \frac{1.003}{2 \times 3.14 \times 50} = \frac{1.003}{314} = 0.0031944$$

$$t = 3.19 \times 10^{-3}\ second$$

Numerical Problems:

1) **In a Young's double slit experiment distance between the slits is $1mm$. The fringe width is found to be $0.6mm$. When the screen is moved through a distance of $0.25m$ away from the plane of the slit, the fringe width becomes $0.75mm$. Find the wavelength of the light used.**

Given: $d = 1\ mm = 1 \times 10^{-3}\ m$
$\beta = 0.6\ mm = 0.6 \times 10^{-3}\ m$
$D = x$
$D' = (x + 0.25)\ m$
$\beta' = 0.75\ mm = 0.75 \times 10^{-3}\ m$

$$\beta = \frac{\lambda D}{d} \quad ----(1)$$
$$\beta' = \frac{\lambda D'}{d} \quad ----(2)$$

Taking equation(2) ÷ (1)
$$\frac{\beta'}{\beta} = \frac{\lambda D'}{d} \times \frac{d}{\lambda D}$$
$$\frac{\beta'}{\beta} = \frac{D'}{D}$$
$$\frac{0.75 \times 10^{-3}}{0.6 \times 10^{-3}} = \frac{(x + 0.25)}{x}$$
$$0.75x = 0.6x + 0.6 \times 0.25 \Longrightarrow (0.75 - 0.6)x = 0.15$$
$$0.15\ x = 0.15 \Longrightarrow x = 1\ m$$

$$\lambda = \frac{\beta d}{D} = \frac{0.6 \times 10^{-3} \times 1 \times 10^{-3}}{1} = 0.6 \times 10^{-6} = 600 \times 10^{-9} = 600\ nm$$

2) **In young's double slit experiment while using a source of light of wavelength 4500 Å, the fringe width is 5 mm. If the distance between the screen and the plane of the slit is reduced to half, what should be the wavelength of light to get fringe width 4 mm?**

Given: $\lambda = 4500\ \text{Å} = 4500 \times 10^{-10}\ m$
$\beta = 5\ mm = 5 \times 10^{-3}\ m$
$D = x$
$D' = \frac{x}{2}$
$\beta' = 4\ mm = 4 \times 10^{-3}\ m$

$$\beta = \frac{\lambda D}{d} \quad ----(1)$$
$$\beta' = \frac{\lambda' D'}{d} \quad ----(2)$$

Taking equation(2) ÷ (1)
$$\frac{\beta'}{\beta} = \frac{\lambda' D'}{d} \times \frac{d}{\lambda D}$$
$$\frac{\beta'}{\beta} = \frac{\lambda' D'}{\lambda D}$$

$$\lambda' = \frac{\beta'\lambda D}{\beta D'} = \frac{4\times10^{-3}\times4500\times10^{-10}\times x}{5\times10^{-3}\times\frac{x}{2}} = \frac{4\times45\times10^{-8}\times2}{5} = 72\times10^{-8} = 720\times10^{-9}$$

$$\lambda' = 720\ nm$$

3) **In Young's double slit experiment, fringes of certain width are produced on the screen kept at a distance from the slits. When the screen is moved away from the slits by 0.1m, fringe width increases by $6\times10^{-5}m$. The separation between the slits is 1mm. Calculate the wavelength of the light used.**

Given: $\beta = x$

$D = y$

$D' = (y+0.1)$

$\beta' = (x + 6\times10^{-5})$

$d = 1\ mm = 1\times10^{-3}\ m$

$$\beta = \frac{\lambda D}{d} \quad ----(1)$$

$$\beta' = \frac{\lambda D'}{d} \quad ----(2)$$

Taking equation$(2)-(1)$

$$\beta' - \beta = \frac{\lambda D'}{d} - \frac{\lambda D}{d} = \frac{\lambda}{d}(D' - D)$$

$$\beta' - \beta = \frac{\lambda}{d}(D' - D)$$

$$x + 6\times10^{-5} - x = \frac{\lambda(y + 0.1 - y)}{1\times10^{-3}}$$

$$\lambda = \frac{6\times10^{-5}\times1\times10^{-3}}{0.1} = 60\times10^{-8} = 600\ nm$$

4) **Monochromatic light of wavelength 5000Å from a narrow slit is incident on the double slit. If the separation of *10* fringes on the screen *1 m* away is *2 cm*. Find the slit separation.**

Given: $\lambda = 5000\ \text{Å} = 5000\times10^{-10}\ m$

$x_{10} = 2\ cm = 2\times10^{-2}\ m$

$D = 1\ m$

β is the fringe width of one bright or dark fringe.

For 10 fringes, 10 fringe width, $x_{10} = 10\ \beta$

$$x_n = n\beta = n\ \frac{\lambda D}{d}$$

$$x_{10} = 10\ \beta = 10\times\frac{\lambda D}{d}$$

$$d = \frac{n\lambda D}{x_{10}} = \frac{10\times5000\times10^{-10}\times1}{2\times10^{-2}} = 2500\times10^{-7} = 0.25\ mm$$

5) **In a Young's double slit experiment, distance between the slits is $0.5\ mm$. When the screen is kept at a distance of $100\ cm$ from the slits. The distance of ninth Bright fringe from the center of the fringe system $8.835\ mm$. Find the wavelength of the light used.**

Given: $d = 0.5\ mm = 0.5 \times 10^{-3}\ m$

$x_9 = 8.835\ mm = 8.835 \times 10^{-3}\ m$

$D = 100\ cm = 1\ m$

Distance of n^{th} bright fringe and central bright fringe is given by,

$$x_n = n\beta = n\,\frac{\lambda D}{d}$$

$$x_9 = 9\,\beta = 9 \times \frac{\lambda D}{d}$$

$$\lambda = \frac{x_9\, d}{9D} = \frac{8.835 \times 10^{-3} \times 0.5 \times 10^{-3}}{9 \times 1} = 0.4908 \times 10^{-6} = 490.8\ nm$$

6) In a Young's double slit experiment wave length of light used is 5000 Å and distance between the slits is 2 mm, distance of screen from the slits is 1m. Find fringe width and also calculate the distance of 7th dark fringe from central bright fringe.

Given: $\lambda = 5000\ \text{Å} = 5000 \times 10^{-10}\ m$

$d = 2\ mm = 2 \times 10^{-3}\ m$

$D = 1\ m$

$$\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{2 \times 10^{-3}} = 2500 \times 10^{-7} = 0.25\ mm$$

Distance of n^{th} dark fringe and central bright fringe is given by,

$$x_n = \left(n - \frac{1}{2}\right)\beta = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}$$

$$x_7 = \left(7 - \frac{1}{2}\right)\beta = \left(7 - \frac{1}{2}\right) \times 2500 \times 10^{-7} = 6.5 \times 2500 \times 10^{-7}$$

$$x_7 = 16250 \times 10^{-7} = 1.625\ mm$$

7) In Young's double slit experiment the slits are separated by $\boldsymbol{0.28\ mm}$ and the screen is placed at a distance of $\boldsymbol{1.4\ m}$ away from the slits. The distance between the central bright fringe and the fifth dark fringe is measured to be $\boldsymbol{1.35\ cm}$. Calculate the wave length of the light used. Also find the fringe width if the screen is moved $\boldsymbol{0.4\ m}$ towards the slits, for the same experimental set up.

Given: $d = 0.28\ mm = 0.28 \times 10^{-3}\ m$

$D = 1.4\ m$

$x_5 = 1.35\ cm = 1.35 \times 10^{-2}\ m$

$D' = 1.4 - 0.4 = 1\ m$

For dark fringe,

$$x_n = \left(n - \frac{1}{2}\right)\beta = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}$$

$$1.35 \times 10^{-2} = \left(5 - \frac{1}{2}\right) \times \frac{\lambda \times 1.4}{0.28 \times 10^{-3}} = \frac{4.5 \times \lambda \times 1.4}{0.28 \times 10^{-3}}$$

$$\lambda = \frac{1.35 \times 10^{-2} \times 0.28 \times 10^{-3}}{4.5 \times 1.4} = \frac{0.378 \times 10^{-5}}{6.3} = 0.06 \times 10^{-5} = 600\ nm$$

$$\beta' = \frac{\lambda D'}{d} = \frac{0.06 \times 10^{-5} \times 1}{0.28 \times 10^{-3}} = 0.2142 \times 10^{-2} = 2.142\ mm$$

8) **Calculate the distance between the centres of fourth bright fringe and seventh dark fringe in an interference pattern produced by Young's double slit experiment of slit separation $\mathbf{1.1}\boldsymbol{mm}$ and separation between the slit and screen being $\mathbf{1.3}\boldsymbol{m}$. Wavelength of the light used is $\mathbf{589.3}\boldsymbol{nm}$.**

Given: $\lambda = 589.3\ nm = 589.3 \times 10^{-9}\ m$

$d = 1.1\ mm = 1.1 \times 10^{-3}\ m$

$D = 1.3\ m$

$$\beta = \frac{\lambda D}{d} = \frac{589.3 \times 10^{-9} \times 1.3}{1.1 \times 10^{-3}} = 696.45 \times 10^{-6} = 0.6964\ mm$$

For dark fringe, (Seventh fringe is dark)

$$x_n = \left(n - \frac{1}{2}\right)\beta$$

$$x_7 = \left(7 - \frac{1}{2}\right)\beta = \left(7 - \frac{1}{2}\right) \times 696.45 \times 10^{-6} = 6.5 \times 696.45 \times 10^{-6}$$

$$x_7 = 4.5268 \times 10^{-3} = 4.5268\ mm$$

For bright fringe, (Fourth fringe is bright)

$$x_n = n\beta$$

$$x_4 = 4\beta = 4 \times 696.45 \times 10^{-6} = 2785.8 \times 10^{-6} = 2.7858\ mm$$

$$x_7 - x_4 = 4.5268 - 2.7858 = 1.741\ mm$$

9) **A beam of light consisting of two wavelengths $\mathbf{4200}$Å and $\mathbf{5600}$Å is used to obtain interference fringes in Young's double slit experiment. The distance between the slits is $\mathbf{0.3}\boldsymbol{mm}$ and the distance between the slit and the screen is $\mathbf{1.5}\boldsymbol{m}$. Compute the least distance of the point from the central maximum, where the bright fringes due to both the wavelengths coincide.**

Given: $\lambda_1 = 4200\ \text{Å} = 4200 \times 10^{-10}\ m$

$\lambda_2 = 5600\ \text{Å} = 5600 \times 10^{-10}\ m$

$d = 0.3\ mm = 0.3 \times 10^{-3}\ m$

$D = 1.5\ m$

We have to calculate the minimum distance at which one of the bright fringe of the first pattern will coincide with the bright fringe of the other pattern from the central maximum.

For bright fringe,

$$x_n = n\beta = n\ \frac{\lambda D}{d}$$

$$For\ \ \lambda_1 \Longrightarrow x_1 = n_1\ \frac{\lambda_1 D}{d}$$

$$For\ \ \lambda_2 \Longrightarrow x_2 = n_2\ \frac{\lambda_2 D}{d}$$

As the both bright fringes coincide, $x_1 = x_2$

$$n_1\ \frac{\lambda_1 D}{d} = n_2\ \frac{\lambda_2 D}{d}$$

$n_1\lambda_1 = n_2\lambda_2$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{5600 \times 10^{-10}}{4200 \times 10^{-10}} = \frac{4}{3}$

$\Longrightarrow n_1 = 4$ and $n_2 = 3$

Fourth bright of the first pattern will coincide with third bright of the other.

10) Light of wavelength 5000Å falls on a plane reflecting surface. What are the wavelength and frequency of reflected light? For what angle of incidence is the reflected ray normal to the incident ray? Speed of light in vacuum $= 3 \times 10^8 ms^{-1}$.

Given: $\lambda = 5000\text{ Å} = 5000 \times 10^{-10}\ m$

$C = 3 \times 10^8\ ms^{-1}$

During reflection, wavelength and frequency will not change.

$\lambda = 5000\text{ Å} = 5000 \times 10^{-10}\ m$

$\nu = \frac{C}{\lambda} = \frac{3 \times 10^8}{5000 \times 10^{-10}} = 0.6 \times 10^{15} = 6 \times 10^{14}\ Hz$

For reflection, $i = r$

$i + r = 90^0$

$i + i = 90^0$

$2\ i = 90^0$

$i = 45^0$

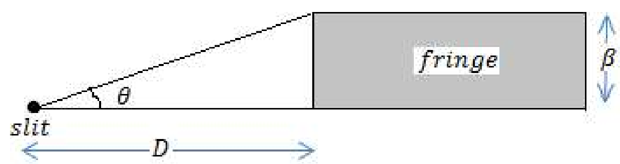
11) In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

Given: $\theta = 0.2^0$

$D = 1\ m$

$n_w = {}^4/_3$

$\lambda = 600\ nm = 600 \times 10^{-9}\ m$



$\tan\theta = \frac{\beta}{D}$

For θ is small $\tan\theta \approx \theta$

$\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}\ ----(1)$

When the apparatus is immersed in water wavelength of light changes from λ to λ'

$\theta' = \frac{\lambda'}{d}$ where $\lambda' = \frac{\lambda}{n_w}$

$\theta' = \frac{\lambda}{n_w d}\ ----(2)$

$\frac{\theta'}{\theta} = \frac{\lambda}{n_w d} \times \frac{d}{\lambda} = \frac{1}{n_w} = \frac{1}{({}^4/_3)} = \frac{3}{4}$

$\theta' = \theta \times \frac{3}{4} = 0.2 \times \frac{3}{4} = 0.15^0$