

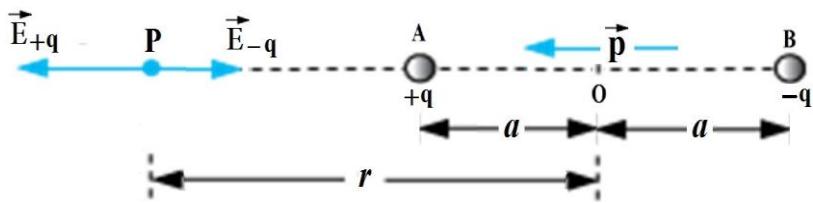
IMPORTANT 5 MARK QUESTIONS AND ANSWERS**Sub : Physics****Class : II PUC****1. Derive the expression for Electric Field at a point on the axial line of an electric dipole.**

Ans: Consider an electric dipole consisting charges $+q$ and $-q$ separated by a distance $2a$ as shown in fig.

Let,

r - distance of point P on the axis from the centre of the dipole on the side of charge $+q$.

$p = q \times 2a$, dipole moment



The magnitude of electric field at P, due to charges $+q$ and $-q$ of dipole is given by

$$E_{+q} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \quad \text{and} \quad E_{-q} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$$

Since \vec{E}_{+q} and \vec{E}_{-q} are in opposite direction, therefore net electric field at P is

$$E = E_{+q} - E_{-q}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$$

$$E = \frac{q}{4\pi\epsilon_0} \frac{(r+a)^2 - (r-a)^2}{(r+a)^2(r-a)^2} = \frac{q}{4\pi\epsilon_0} \frac{4ar}{(r^2 - a^2)^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{4qar}{(r^2 - a^2)^2} \quad \text{and net electric field is along the direction of } \vec{p}.$$

$$\text{OR} \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}r}{(r^2 - a^2)^2} \hat{p} \quad \text{where, } p = q(2a)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}r}{(r^2 - a^2)^2}$$

* For $r \gg a$,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}$$

2.**Derive the expression for Electric Field at a point on the equatorial plane of an electric dipole.**

Ans: Consider an electric dipole consisting charges $+q$

and $-q$ separated by a distance $2a$ as shown in fig.

Let,

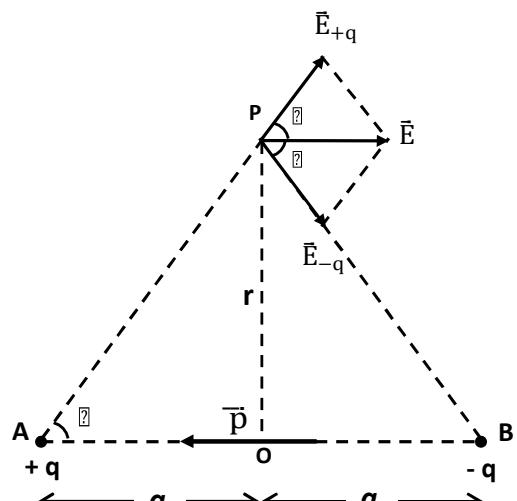
$$p = q \times 2a, \text{ - dipole moment}$$

r - distance of point P on equatorial line from the center of dipole.

The magnitudes of the electric field at P, due to the two charges $+q$ and $-q$ are given by,

$$E_{+q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2} \quad \text{and} \quad E_{-q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2 + a^2} \quad \dots \dots \dots (1)$$

$$\therefore E_{+q} = E_{-q} \quad \dots \dots \dots (2)$$



(i) The components of fields \vec{E}_{+q} and \vec{E}_{-q} , normal to the dipole axis ($E_{+q} \sin \theta$ and $E_{-q} \sin \theta$) equal and opposite and hence they get cancel.

(ii) The field components along the dipole axis ($E_{+q} \cos \theta$ and $E_{-q} \cos \theta$) are get add up.

∴ Net electric field at P is, $E = (E_{+q} \cos \theta + E_{-q} \cos \theta)$

$$E = (E_{+q} + E_{-q}) \cos \theta = \frac{2E_{+q} \cos \theta}{a}$$

From figure, $\cos \theta = \frac{a}{(r^2 + a^2)^{1/2}}$ and using equation (1)

$$\therefore E = \left[\frac{1}{4\pi\epsilon_0} \frac{2q}{(r^2 + a^2)} \right] \frac{a}{(r^2 + a^2)^{1/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{2qa}{(r^2 + a^2)^{3/2}}$$

and is along $(-\vec{p})$

$$\text{OR } \vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2 + a^2)^{3/2}}$$

$$* \text{ For } r \gg a, \quad \vec{E} = \frac{1}{4\pi\epsilon_0 r^3} \vec{p}$$

3. Obtain the expression for electric field at an outside point due to a uniformly charged spherical shell.

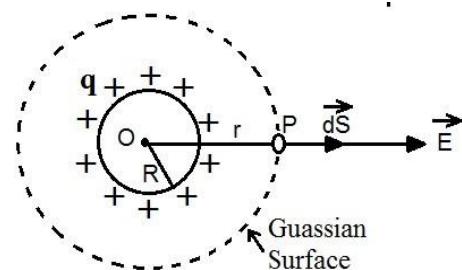
Ans: Consider a uniformly charged spherical shell as shown in figure.

Let,

R - radius of uniformly charged spherical shell

q - total charge on the shell

r - radius of the Gaussian sphere



By the spherical symmetry electric intensity 'E' is along the radius and is same at all points on the surface. Further, at every point on the Gaussian surface, angle between \vec{E} and area element $d\vec{s}$ is zero.

Therefore, electric flux passing through the Gaussian surface is,

$$\phi = \sum E ds \cos \theta = \sum E ds \cos 0^\circ = \sum E ds$$

$$\phi = E \sum ds$$

But $\sum ds = 4\pi r^2$, the surface area of the spherical shell.

$$\therefore \phi = E [4\pi r^2] \dots \dots \dots (1)$$

From Gauss's law, the total electric flux passing through the Gaussian surface is

$$\phi = \frac{q}{\epsilon_0} \dots \dots \dots (2)$$

From Equation (1) and (2)

$$E [4\pi r^2] = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r^2} \right)$$

From the above equation. It is clear that electric intensity at a point outside the sphere is the same as if the entire charge were concentrated at the center of the shell.

{For the point P inside the shell at a distance r ($< R$) from the center O, the Gaussian surface do not enclose any charge. $\therefore \sum q = 0$

$$\therefore \phi = \frac{q}{\epsilon_0} = 0 \quad \Rightarrow \phi = E(4\pi R^2) = 0 \quad \Rightarrow E = 0$$

i.e. Electric field inside a charged conductor is "zero"}

4. Obtain the expression for electrical potential at a point due to a point charge.

Ans: Consider a charge at $+q$ located at origin O in free space as shown in

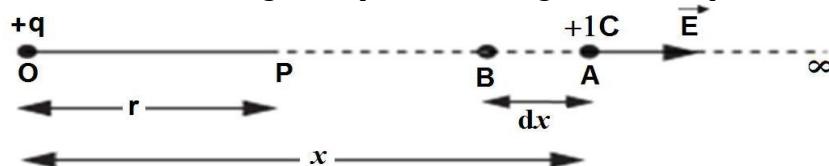


figure.

The electric potential at point P is the amount of work done in carrying a unit positive charge from ∞ to the point P.

Let 'A' be an intermediate point on this path, small amount of work done in moving $+1\text{ C}$ through a distance ' dx ' from A to B

$$dW = \bar{F} \cdot \bar{dx} = F dx \cos 180^\circ = -F dx$$

$$\text{But the force on } +1\text{ C} \text{ at A is, } F = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

$$\therefore dW = -\frac{1}{4\pi\epsilon_0} \frac{q}{x^2} dx$$

\therefore Total work done in moving unit positive ($+1\text{ C}$) charge from ∞ to point P is

$$\int dW = \int_{\infty}^r -\frac{1}{4\pi\epsilon_0} \frac{q}{x^2} dx$$

$$W = \frac{q}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{x^2} dx = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^r$$

$$W = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

By the definition, this work done is the potential at P due to the charge at O.

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

5.

Derive the expression for capacitance of parallel plate capacitor.

Ans: A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance.

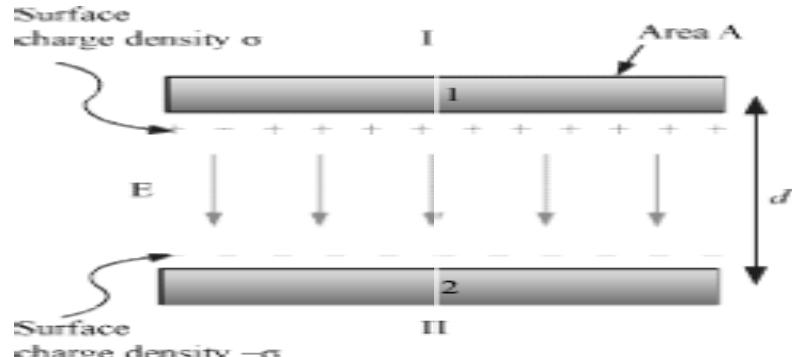
Let,

A - area of each plate

d - distance of separation b/w plates

Q - charge on each plate of capacitor

$\sigma = Q/A$ - surface charge density



Electric field in the outer region I and II is zero.

In the inner region between the plates 1 and 2, the electric field is,

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \quad \text{--- (1)}$$

The potential difference between the plates of capacitor is given by,

$$V = Ed = \frac{\sigma}{\epsilon_0} d = \frac{Qd}{\epsilon_0 A} \quad \text{--- (2)}$$

The Capacitance C of the parallel plate capacitor is given by,

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

6. Obtain the expression for energy stored in the charged capacitor. (3/5 M)

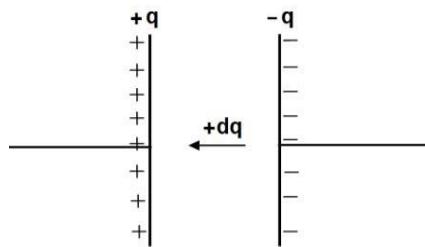
Ans: The energy stored in the charged capacitor is the total work done in charging the capacitor to a given potential, by transferring charges from one plate to another plate of the capacitor.

Consider an intermediate situation of charging the capacitor,

Let $q \rightarrow$ total charge on capacitor at the intermediate situation and

$V \rightarrow$ potential difference between the two plates of capacitor so that

$$V' = \frac{q}{C} \quad \dots \dots \dots (1)$$



Now, the small amount of work done in transferring an additional charge dq from the negative plate to the positive plate is given by,

$$dW = V' dq = \frac{q}{C} dq$$

Therefore the total work done in transferring charge from 0 to Q is given by,

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{1}{C} \left[\frac{Q^2}{2} - 0 \right]$$

$$W = \frac{Q^2}{2C}$$

This work is stored as electrostatic potential energy U in the capacitor.

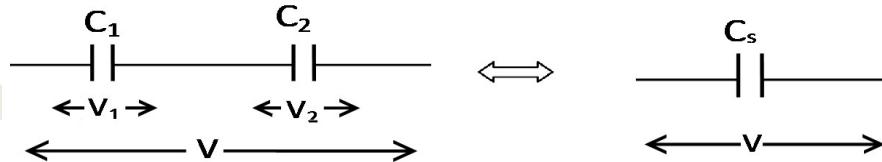
$$\therefore U = \frac{Q^2}{2C}$$

$$* \text{ Also, } U = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

7.

Derive the expression for equivalent capacitance of two capacitors connected in series. (3/5 M)

Ans: Consider two capacitors C_1 and C_2 are connected in series across a potential difference V , as shown in fig.



In series combination of capacitors, the charge on each capacitor is same. Let Q be the charge on each capacitor.

The potential difference applied across their combination is the sum of the potential differences across each capacitor.

$$\therefore V = V_1 + V_2$$

$$\text{but } V_1 = \frac{Q}{C_1} \text{ and } V_2 = \frac{Q}{C_2}$$

$$\therefore V = \frac{Q}{C_1} + \frac{Q}{C_2} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \quad \dots \dots \dots (1)$$

For equivalent capacitor of capacitance C_s , under same applied potential difference V volts,

$$V = \frac{Q}{C_s} \quad \dots \dots \dots (2)$$

Combining (1) and (2), we obtain

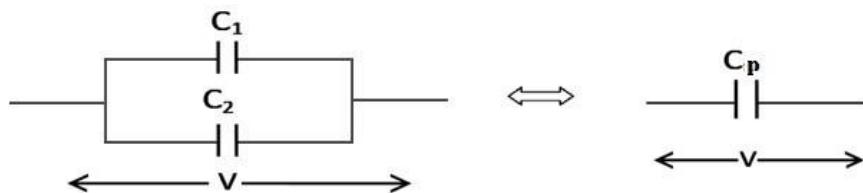
$$\frac{Q}{C_s} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

8. Derive the expression for equivalent capacitance of two capacitors connected in parallel.

(3/5 M)

Ans: Consider two capacitors C_1 and C_2 are connected in parallel across a potential difference V , as shown in fig.



In parallel combination of capacitors, the potential difference across each capacitor is same and is same as that of applied potential V .

The total charge stored in the combination is the sum of the charges on each capacitor.

$$\therefore Q = Q_1 + Q_2$$

$$\text{but } Q_1 = C_1 V \text{ and } Q_2 = C_2 V$$

$$\therefore Q = C_1 V + C_2 V = (C_1 + C_2)V \quad \dots \dots \dots (1)$$

For equivalent capacitor of capacitance C_p , under same applied potential difference V volt,

$$Q = C_p V \quad \dots \dots \dots (2)$$

From equation (1) and (2), we have

$$C_p V = (C_1 + C_2)V$$

$$C_p = C_1 + C_2$$

9.

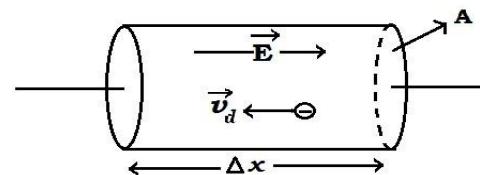
Derive the expression for current in a conductor in terms of drift velocity. And hence arrive at the expression for electrical conductivity of material of a conductor. (Assume the expression for drift velocity)

Ans: Consider a conductor carrying steady current.

Let, Δx - length of a element of a conductor

A - uniform cross sectional area of conductor

n - number density of free electrons in the conductor
(free electron density)



Total number of free electrons in the element is, $N = (\text{Charge density}) (\text{volume}) = n(A\Delta x)$

Magnitude of charge due to these electrons is $\Delta q = (nA\Delta x) e \dots \dots \dots (1)$

Where, e - charge of electron.

If Δt is the time taken by this charge to pass through the element of conductor, then current through the conductor is

$$I = \frac{\Delta q}{\Delta t}$$

$$I = \frac{nA\Delta x e}{\Delta t} = nAe \left(\frac{\Delta x}{\Delta t} \right)$$

But $\frac{\Delta x}{\Delta t} = v_d$ the drift velocity (magnitude) of conduction electrons

$$\therefore I = nAe v_d \quad \dots \dots \dots (2)$$

$$eE$$

Magnitude of drift velocity is $v_d = \frac{eE}{N} v$

Where, E - electric field in the conductor, m is mass of electron & τ is relaxation time.

$$I = nAe \left(\frac{eE}{N} v \right)$$

$$\frac{I}{A} = \frac{ne^2 v}{N} E \quad \text{but current density } j = \frac{I}{A}$$

$$\therefore j = \frac{ne^2 v}{N} E \quad \dots \dots \dots (3)$$

From Ohm's law (vector form) $j = \sigma E \quad \dots \dots \dots (4)$

Where, σ is electrical conductivity.

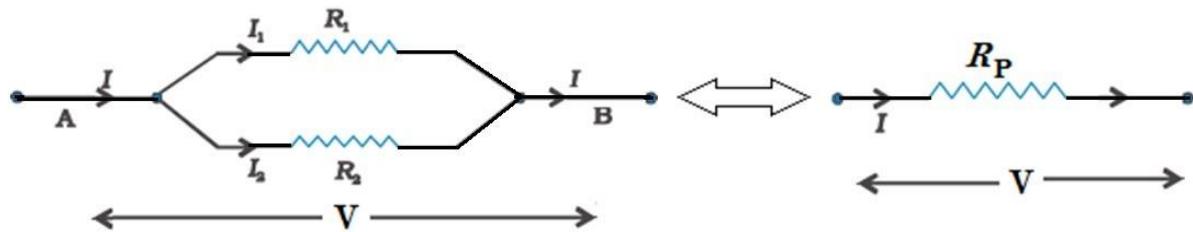
From equation (3) and (4) we get

$$o = \frac{ne^2v}{m}$$

- Electrical resistivity, $o = \frac{1}{R} = \frac{N}{ne^2v}$

10. What is equivalent resistance? And obtain the expression for effective resistance of two resistors connected in parallel. (3/5 M)

Ans: A single resistance which produces same effect (allows same current) as the combination of resistances under the similar conditions (same potential difference) is called equivalent resistance.



Consider two resistors of resistance R_1 and R_2 connected in parallel across a potential difference of V volts. In parallel combination, the potential difference (V) across each resistor is same. Let the steady current I in the circuit divide into I_1 and I_2 through the resistors R_1 and R_2 respectively.

Hence, $I = I_1 + I_2$

From Ohm's law: $I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$

$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad \text{--- (1)}$$

For the equivalent circuit of resistance R_p , under same potential difference V volt,

$$I = \frac{V}{R_p} \quad \text{--- (2)}$$

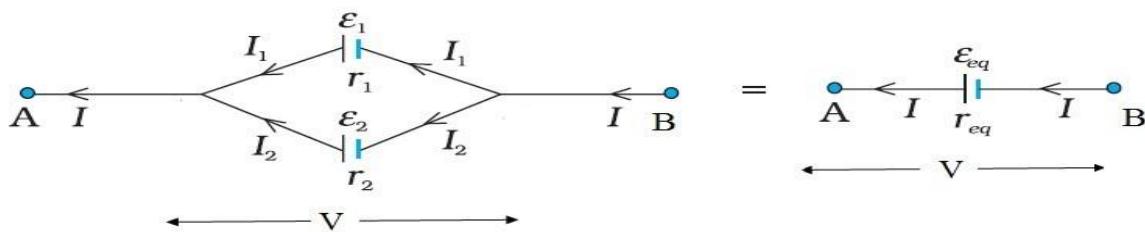
From equations (1) and (2)

$$\frac{V}{R_p} = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

11. Derive the expressions for equivalent emf and equivalent internal resistance of parallel combination of two cells.

Ans: Consider two sources of emf (cells) connected in parallel as shown in figure.



Let, ϵ_1 and ϵ_2 - emf of two cells

r_1, r_2 - Internal resistance of two cells

I_1 and I_2 - Currents through the branches of cells ϵ_1 and ϵ_2 respectively

I - Net current in the branch AB,

The total current due to this combination of cells is

$$I = I_1 + I_2 \quad \text{--- (1)}$$

Terminal potential difference across the first cell, $V = \epsilon_1 - I_1 r_1$

$$\Rightarrow I_1 = \frac{\epsilon_1 - V}{r_1}$$

Terminal potential difference across the second cell, $V = \varepsilon_2 - I_2 r_2$

$$\Rightarrow I_2 = \frac{\varepsilon_2 - V}{r_2}$$

$$\therefore I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad \dots \dots \dots (2)$$

If the combination of cells is replaced by an equivalent cell of emf ε_{eq} and internal resistance r_{eq} , then terminal potential difference of that cell is

$$V = \varepsilon_{eq} - Ir_{eq}$$

$$I = \frac{\varepsilon_{eq}}{r_{eq}} - V \left(\frac{1}{r_{eq}} \right) \quad \dots \dots \dots (3)$$

From equation (2) and (3)

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \text{and} \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

12. Deduce the balancing condition for Wheatstone's network using Kirchhoff's rules.

Ans: If the current through the galvanometer is zero ($I_g = 0$), the network is said to be balanced. In this case, the galvanometer shows no deflection.

Consider the Wheatstone's network as shown in figure.

Condition for balance:

At balanced state of network i.e. $I_g = 0$,

Applying Kirchhoff's node rule for nodes B and D, we get

$$I_1 = I_3 \quad \text{and} \quad I_2 = I_4 \quad \dots \dots \dots (1)$$

Applying Kirchhoff's second law to the mesh ABDA,

$$I_1 P + 0 - I_2 R = 0$$

$$\Rightarrow I_1 P = I_2 R \quad \dots \dots \dots (2)$$

Applying Kirchhoff's second law to the mesh BCDB,

$$I_3 Q - I_4 S - 0 = 0$$

$$I_3 Q = I_4 S \quad \dots \dots \dots (3)$$

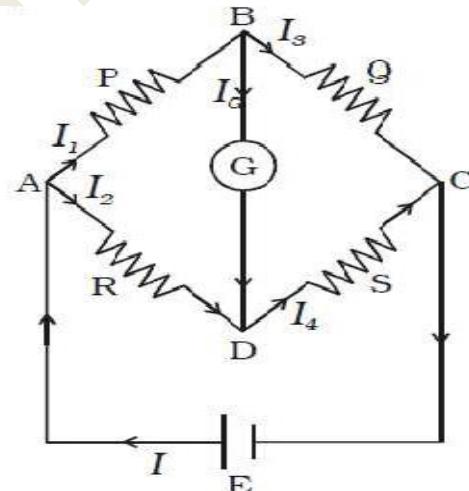
Dividing equation (2) by (3)

$$\frac{I_1 P}{I_3 Q} = \frac{I_2 R}{I_4 S}$$

Using equation (1), the above equation changes to

$$\frac{P}{Q} = \frac{R}{S}$$

This is the condition for balance of Wheatstone network.



13. Derive the expression for magnetic field at a point on the axis of a circular current loop.

Ans: Consider a circular coil carrying current as shown in figure.

Let,

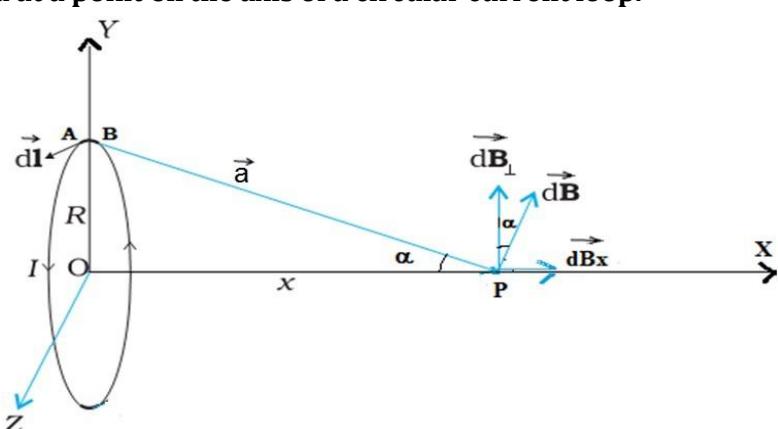
R - radius of current loop

I - current in the loop

dl - length of current element AB

x - distance of point P on the axis

from the center of current loop O



Magnetic field at P due to current element 'AB' of length 'dl' is

$$dB = \frac{\mu_0}{4n} \frac{Idl \sin \theta}{a^2}$$

Here, $\theta = 90^\circ$, $\sin 90^\circ = 1$ and $a^2 = R^2 + x^2$

$$\therefore dB = \frac{\mu_0}{4n} \frac{Idl}{(R^2 + x^2)} \quad \dots \dots \dots (1)$$

This magnetic field $d\vec{B}$ can be resolved into two components, $dB_T = dB \cos \alpha$ and $dB_s = dB \sin \alpha$.

If the magnetic field at P is summed over the entire loop,

(a) all the perpendicular components dB_T are cancelled out and

(b) the components dB_s adds up.

Hence the magnetic field at P due to entire current loop is

$$B = \sum_s dB_s = \sum_s dB \sin \alpha = \sum_s \left(\frac{\mu_0}{4n} \frac{Idl}{(R^2 + x^2)} \right) \sin \alpha$$

$$B = \sum_s \left(\frac{\mu_0}{4n} \frac{Idl}{(R^2 + x^2)} \right) \frac{R}{(R^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0}{4n} \frac{IR}{(R^2 + x^2)^{3/2}} \quad (\sum dl) = \frac{\mu_0}{4n} \frac{IR}{(R^2 + x^2)^2} \quad (2nR) \quad (\because \text{for circular loop } \sum dl = 2nR)$$

$$B = \frac{\mu_0}{4G} \frac{2GR^2I}{(R^2 + x^2)^{3/2}}$$

$$(\because \text{from fig. } \sin \alpha = \frac{R}{(R^2 + x^2)^{1/2}})$$

$$(\because \text{for circular loop } \sum dl = 2nR)$$

14. Derive the expression for the force between two infinitely long straight parallel conductors carrying currents and hence define ampere.

Ans: Consider two infinitely long straight parallel conductors a and b carrying currents I_1 and I_2 respectively and separated by a perpendicular distance 'd' as shown in the figure.

The magnetic field at each point on conductor 'b' due to current I_1 in conductor 'a' is

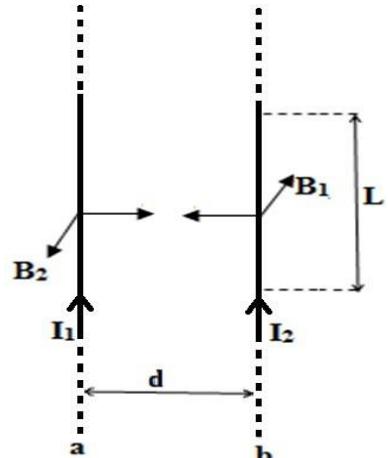
$$B_1 = \frac{\mu_0}{4n} \frac{2I_1}{d}$$

Now the current carrying conductor 'b' is in uniform magnetic field B_1 . Hence magnetic force on the segment L of conductor 'b' is

$$F_1 = I_2 L B_1 \sin \theta$$

F_1 is directed towards the conductor 'a' and here $\theta = 90^\circ$, $\sin 90^\circ = 1$

$$F_1 = \frac{\mu_0}{4n} \frac{2I_1 I_2}{d} L$$



Similarly conductor 'a' also experience same magnitude of force but in opposite direction. Magnetic force on segment L of conductor 'a' is

$$F_2 = \frac{\mu_0}{4n} \frac{2I_1 I_2}{d} L$$

The mutual force per unit length on conductors 'a' and 'b' is, $F = \frac{F_1}{L}$ or $F = \frac{F_2}{L}$

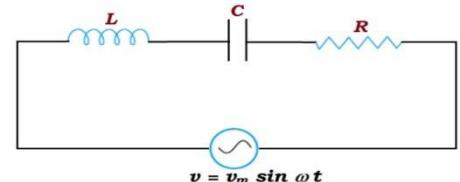
$$\therefore F = \frac{\mu_0}{4G} \frac{2I_1 I_2}{d}$$

Definition of ampere: Thus "If two very long, straight, parallel conductors of negligible cross section carrying same steady current are placed 1 m apart in free space (vacuum) experience a mutual force of 2×10^{-7} newton per meter length of these conductors, then the current in each conductor is said to be 1 A".

15. Using phasor diagram, derive the expression for current in the series LCR circuit in terms of impedance Z and phase difference ϕ .

Ans: Consider a series LCR circuit connected to an ac source ϵ . Let the voltage of the source to be,

$$v = v_m \sin \omega t \quad \dots \dots \dots (1)$$



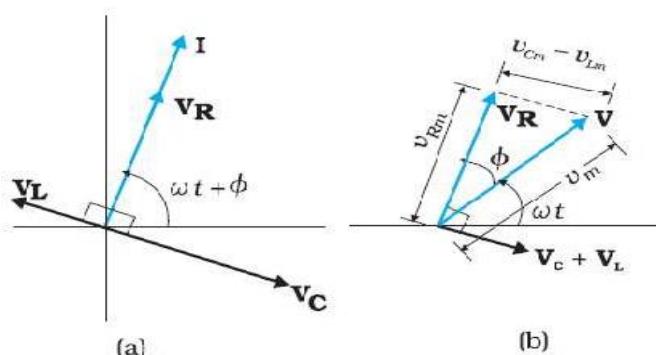
If q is the charge on the capacitor and I the current, at time t , we have, from Kirchhoff's loop rule:

$$v_R + v_C + v_L = v \\ iR + \frac{q}{C} + L \frac{di}{dt} = v_N \sin \omega t \quad (2)$$

Let the current in the circuit be $i = i_N \sin(\omega t + \phi)$ ----- (3)

Where, ϕ - phase difference between voltage across the source and the current in the circuit.

The phasor diagram for circuit at some instant of time t is as shown in figure.



Let,

\bar{I} - phasor representing the current in the circuit

\bar{V}_R - phasor representing voltage across resistor

\bar{V}_L - phasor representing voltage across inductor

\bar{V}_C - phasor representing voltage across capacitor

\bar{V} - phasor representing voltage across the source

The length of these phasors (or amplitude) of \bar{V}_R , \bar{V}_C and \bar{V}_L are:

$$v_{RN} = i_N R, \quad v_{CN} = i_N X_C, \quad v_{LN} = i_N X_L \quad (4)$$

We know that \bar{V}_R is parallel to \bar{I} , \bar{V}_C is $\pi/2$ behind \bar{I} and \bar{V}_L is $\pi/2$ ahead of \bar{I} .

The phasor relation for these voltages is $\bar{V}_L + \bar{V}_R + \bar{V}_C = \bar{V}$ ----- (5)

Since \bar{V}_C and \bar{V}_L are always along the same line and in opposite directions, they can be combined into a single phasor ($\bar{V}_C + \bar{V}_L$), which has a magnitude $|v_{CN} - v_{LN}|$.

Using the Pythagorean Theorem for fig. (b) we have:

$$v^2 = v_{RN}^2 + (v_{CN} - v_{LN})^2$$

Substituting the values of v_{RN} , v_{CN} , and v_{LN} from Eq. (4) into the above equation, we have

$$v^2 = (i_N R)^2 + (i_N X_C - i_N X_L)^2 = i^2 [R^2 + (X_C - X_L)^2]$$

$$\Rightarrow i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{v_m}{Z} \quad (6)$$

Where, $Z = \sqrt{R^2 + (X_C - X_L)^2}$ ----- (7) is called the impedance of the ac circuit.

Since phasor \bar{I} is always parallel to phasor \bar{V}_R , the phase angle ϕ is the angle between \bar{V}_R and \bar{V} and can be determined from Fig. (b):

$$\tan \phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}} = \frac{X_C - X_L}{R} \quad (8)$$

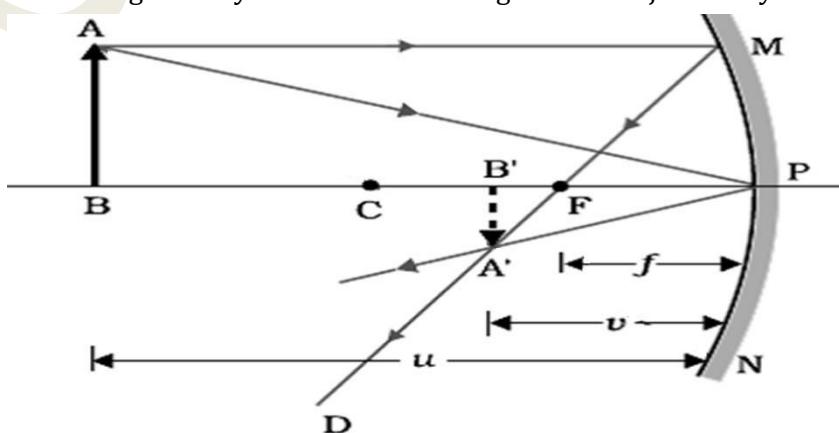
Equation (6) gives the amplitude of the current and Eq. (8) gives the phase angle. With these, Eq. (3) is completely specifies the current in the circuit.

16.

Derive the Mirror formula.

OR Obtain the relation between object distance (u), image distance (v) and the focal length (f) of a spherical mirror.

Ans: The geometry of formation of image $B'A'$ of object BA by a concave mirror is as shown in figure.



MPN = spherical mirror,
AB = linear size of the object,

A'B' = linear size of the image,

BP = u = object distance

B'P = v = image distance

FP = f = focal length

CP = R = radius of curvature

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = (n_{21} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (\because n_{21} = \frac{n_2}{n_1})$$

Suppose the object is at infinity, i.e., $u \rightarrow \infty$ then, $v \rightarrow f$

$$\therefore \frac{1}{f} = (n_{21} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \dots \dots \dots (3)$$

This is lens maker's formula

19. Derive expression for equivalent focal length of the two thin convex lenses in contact with each other.

Ans:

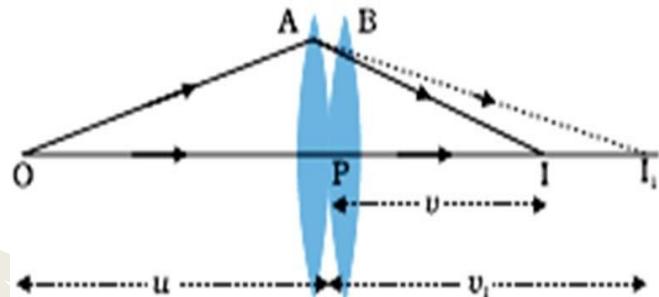
Let,

A and B - two thin convex lenses

f_1 - focal length of lens A

f_2 - focal length of lens B

O - point object placed beyond focus of lens A



The geometry of image formation by combination of lenses is as shown in figure.

The image formation takes place in two stages.

(i) The first lens forms the real image at I_1 of the object O. For the image formed by the first lens A,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \dots \dots \dots (1)$$

(ii) The image I_1 acts as virtual object for the second lens B and the lens B forms final image I. For the image formed by the second lens B,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \dots \dots \dots (2)$$

Adding Equations (1) and (2), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots \dots \dots (3)$$

If the two lens-systems is regarded as equivalent to a single lens of focal length f, then,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \dots \dots \dots (4)$$

From equations (3) and (4)

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots \dots \dots (5)$$

20. Derive the refractive index of material of prism in terms of its refracting angle A and angle of minimum deviation D_m .

Ans: The refraction of a ray of light through prism is as shown in figure.

Let, ABC = principal section of the prism

A = Refracting angle of the prism

PQ = incident ray

QR = refracted ray

RS = emergent ray

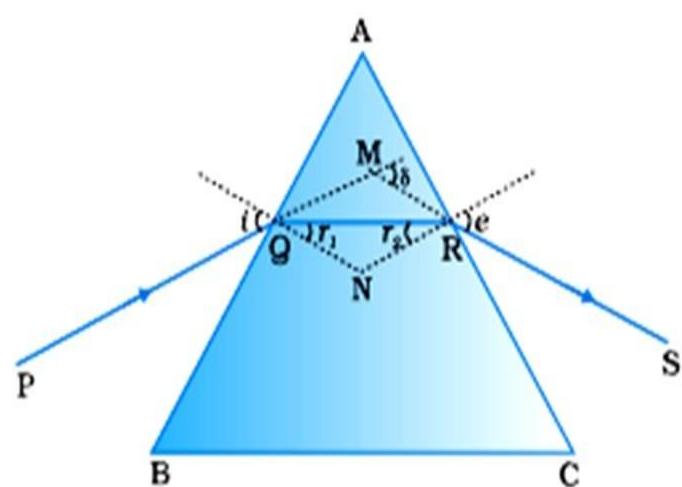
i = angle of incidence

e = angle of emergence

r_1 & r_2 angles of refraction

In the quadrilateral AQNR,

$$\angle A + \angle QNR = 180^\circ$$



From the triangle QNR,

$$r_1 + r_2 + \angle QNR = 180^\circ$$

Comparing these two equations,

$$r_1 + r_2 = A \dots \dots \dots (1)$$

The angle between the emergent ray RS and the direction of the incident ray PQ is called the **angle of deviation** δ . The total deviation δ is equal to the sum of deviations at the two faces,

$$\delta = \delta_1 + \delta_2$$

$$\delta = (i - r_1) + (e - r_2) = i + e - (r_1 + r_2)$$

$$\text{i.e., } \delta = i + e - A \dots \dots \dots (2)$$

A plot between the angle of deviation and angle of incidence is shown in Figure.

At the minimum deviation D_m ,

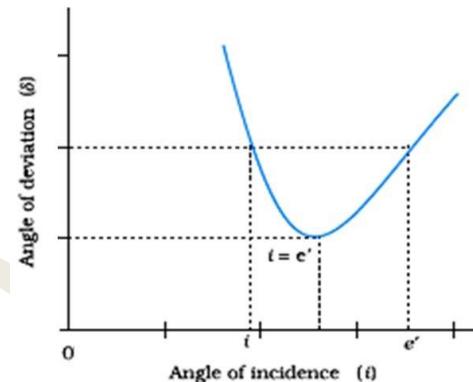
We have, $\delta = D_m$, $i = e$ which implies $r_1 = r_2$

Equation (1) gives, $2r = A$ or $r = A/2$

Equation (2) gives, $D_m = 2i - A$ or $i = \frac{(A + D_m)}{2}$

The refractive index of the prism from Snell's law is

$$n_{21} = \frac{n}{n_1} = \frac{\sin i}{\sin r} = \frac{\sin \left[\frac{(A + D_m)}{2} \right]}{\sin \left[\frac{A}{2} \right]}$$



21. Derive the expression for fringe width of interference fringes using Young's double slit experiment.

Ans: Consider Young's double slit arrangement for obtaining interference fringes as shown in figure.

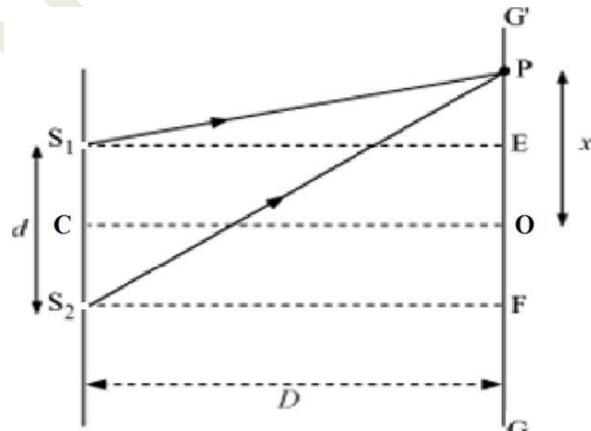
Let,

S_1 and S_2 - two coherent sources (Young's double slits)

d - distance between slits

D - distance of screen from coherent sources/slits

O - center point on the screen and is equidistant from S_1 and S_2



The path difference between the two light waves from S_1 and S_2 reaching the point O is zero. Thus the point O has **maximum intensity**.

Consider a point P at a distance x from O .

The path difference between the light waves from S_1 and S_2 reaching the point P is, $\delta = S_2P - S_1P$

$$\text{From the figure, } (S_2P)^2 = (S_2F)^2 + (FP)^2 = D^2 + \left(x + \frac{d}{2} \right)^2$$

$$\text{Similarly } (S_1P)^2 = (S_1E)^2 + (EP)^2 = D^2 + \left(x - \frac{d}{2} \right)^2$$

$$\therefore (S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right]$$

$$= \left[D^2 + x^2 + \frac{d^2}{4} + 2(x) \left(\frac{d}{2} \right) \right] - \left[D^2 + x^2 + \frac{d^2}{4} - 2(x) \left(\frac{d}{2} \right) \right] = 2xd$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

$$(S_2P - S_1P) = \frac{2}{(S_2P + S_1P)} \cdot 2xd$$

Since P is very close to O and $d \ll D$, therefore $(S_2P + S_1P) \approx 2D$

$$\text{Path difference, } (S_2P - S_1P) = \frac{2x}{2D} = \frac{x}{D} \quad \dots \dots \dots (1)$$

Equation (1) represents the path difference between light waves from S_1 and S_2 superposing at the point P.

For constructive interference, $\frac{x}{D} = n\lambda$ or $x = n\left(\frac{\lambda D}{d}\right)$ where, $n = 0, 1, 2 \dots$

i.e., The distance of the n^{th} bright fringe from the centre O of the screen is $x = n\left(\frac{\lambda D}{d}\right)$

The distance of $(n + 1)^{\text{th}}$ bright fringe from the centre of the screen is $x = (n + 1)\left(\frac{\lambda D}{d}\right)$

The distance between the centers of any two consecutive bright fringes is called the **fringe width** of bright fringes. The **fringe width** is given by,

$$\beta = x_{n+1} - x_n = (n+1)\left(\frac{\lambda D}{d}\right) - n\left(\frac{\lambda D}{d}\right) = \frac{\lambda D}{d}$$

$$\therefore \boxed{\beta = \frac{\lambda D}{d}}$$

Similarly for dark fringes also we obtain the same expression for fringe width.

22.

Write the experimental observations of photoelectric effect. (3/5 M)

Ans: The experimental observations of photoelectric effect are,

- 1) The photoelectric emission is an **instantaneous process**.
- 2) For every photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation below which there is no photoelectric emission. This minimum frequency is called the **threshold frequency**.
- 3) Above threshold frequency, the **photo current is directly proportional to the intensity of incident light**.
- 4) Above the threshold frequency, the **stopping potential or the maximum kinetic energy of the photoelectrons is directly proportional to the frequency of the incident radiation, but is independent of its intensity**.
- 5) Above the threshold frequency, saturation current is proportional to the intensity of incident radiation.
- 6) The photo current decreases with increase in negative potential of collector and reaches zero at a negative potential known as stopping potential.

23.

Write the Einstein's photoelectric equation. Using the equation, explain any two experimental observations of photoelectric effect. (3/5 M)

Ans: Albert radiation energy is built up of discrete units called quanta of energy of radiation. Each quantum of radiation has energy $E = hv$,

Where, h is Planck's constant and v - the frequency of light.

In photoelectric effect, an electron absorbs a quantum of energy (hv) of radiation and the electron is emitted with maximum kinetic energy:

$$K_{\text{max}} = hv - \Phi_0 \quad \text{where, } \Phi_0 - \text{work function}$$

This is known as Einstein's photoelectric equation.

- (i) According to Einstein's photoelectric equation, K_{max} depends linearly on frequency v and K_{max} is independent of intensity of radiation.
- (ii) Since K_{max} must be non-negative, photoelectric emission is possible only if $hv > \Phi_0$ or $v > v_0$. Thus, there exists a threshold frequency v_0 for every metal surface, below which no photoelectric emission is possible.

(iii) Intensity of radiation is proportional to the number of photons per unit area per unit time. The greater the number of photons available, the greater is the number of electrons coming out of the metal.

Therefore, (for frequencies $\nu > \nu_0$) photoelectric current is directly proportional to intensity of incident radiation.

(iv) According to Einstein, the photoelectric effect is instantaneous process. This is because photoelectric effect process involves absorption of light quantum by single electron, which takes place instantaneously.

In this way Einstein's theory successfully explains the experimental observations of photo electric effect.

24. Obtain the expression for radius of nth orbit of H-atom, by using the postulates of Bohr atomic model.

Ans: Consider a atom with effective nuclear charge $+Ze$. Let an electron revolves around the nucleus with speed v in the orbit of radius r as shown in figure.

The necessary centripetal force on electron is provided by the electrostatic force between the electron and the nucleus. Therefore we have,

$$\text{centripetal force} = \text{electrostatic force}$$

$$\frac{Nv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{Ze}{r^2}$$

$$Nv^2r = \frac{Ze^2}{4\pi\epsilon_0} \quad \dots \dots \quad (1)$$

From Bohr's angular momentum quantization rule,

$$\frac{Nvr}{nh}$$

$$N^2v^2r^2 = \frac{n^2h^2}{4n^2} \quad \dots \dots \quad (2)$$

Dividing equation (2) by equation (1) we have,

$$\frac{N^2v^2r^2}{Nv^2r} = \frac{n^2h^2}{4n^2} \times \frac{4\pi\epsilon_0}{Ze^2}$$

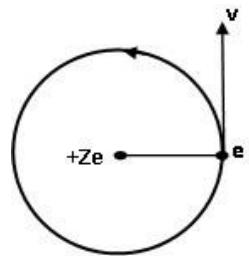
$$\frac{n^2h^2s_0}{Nr} = \frac{n^2h^2s_0}{nZe^2} \Rightarrow r = \frac{n^2h^2s_0}{nNZe^2}$$

For n^{th} orbit,

$$r_n = \frac{n^2h^2s_0}{nNZe^2}$$

For H atom $Z = 1$ and for n^{th} orbit,

$$r_n = \frac{n^2h^2s_0}{nNe^2}$$



25. Obtain the expression for energy of the electron in the nth orbit of H-atom, by using the postulates of Bohr atomic model.

Ans: Consider a atom with effective nuclear charge $+Ze$. Let the electron revolves around the nucleus with speed v in the orbit of radius r as shown in figure.

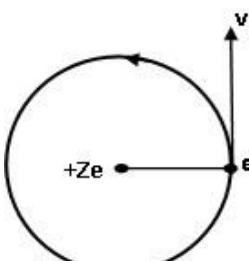
The necessary centripetal force on electron is provided by the electrostatic force between the electron and the nucleus. Therefore we have,

$$\text{centripetal force} = \text{electrostatic force}$$

$$\frac{Nv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{Ze}{r^2}$$

$$\frac{1}{2} Nv^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r}$$

$$KE = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r} \quad \dots \dots \quad (1)$$



PE of the electron = (potential at a distance r from the nucleus) (-e)

$$PE = \left(\frac{1}{4\pi\epsilon_0} \frac{Ze}{r} \right) (-e)$$

$$PE = - \left(\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r} \right) \quad \dots \quad (2)$$

Total energy of the electron in the orbit of radius r is, $E = KE + PE$

$$\therefore E = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r} - \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r} \left(\frac{1}{2} - 1 \right)$$

$$E = - \frac{1}{8\pi\epsilon_0} \frac{Ze^2}{r}$$

The radius of the orbit is given by, $r = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2}$

$$E = - \frac{1}{8\pi\epsilon_0} \frac{Ze^2}{\left(\frac{n^2 h^2 \epsilon_0}{\pi m Z e^2} \right)} = - \frac{1}{8\pi\epsilon_0} Ze^2 \left(\frac{\pi m Z e^2}{n^2 h^2 \epsilon_0} \right)$$

$$E = - \frac{\pi m Z^2 e^4}{8\epsilon_0^2 n^2 h^2}$$

$$\text{For } n^{\text{th}} \text{ orbit, } E_n = - \frac{\pi m Z^2 e^4}{8\epsilon_0^2 n^2 h^2}$$

For hydrogen atom $Z = 1$ & for n^{th} orbit

$$E_n = - \frac{me^4}{8\epsilon_0^2 n^2 h^2}$$

26. Write a note on spectral series of hydrogen atom.

Ans: On the basis transition of electrons between orbits, the radiations emitted are classified into five spectral series as follows.

1. Lyman series: Spectral lines of Lyman series are obtained when the electrons make transitions from higher orbits to the 1^{st} orbit. The wavelength of emitted photon is given by the relation,

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right) \quad n = 2, 3, 4, \dots$$

This series lies in UV region.

2. Balmer series: Spectral lines of Balmer series are obtained when the electrons make transitions from higher orbits to the 2^{nd} orbit. The wavelength of emitted photon is given by the relation,

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, \dots$$

This series lies in Visible region.

3. Paschen series: Spectral lines of Paschen series are obtained when the electrons make transitions from higher orbits to the 3^{rd} orbit. The wavelength of emitted photon is given by the relation,

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right) \quad n = 4, 5, 6, \dots$$

This series lies in near Infrared region.

4. Brackett series: Spectral lines of Brackett series are obtained when the electrons make transitions from higher orbits to the 4^{th} orbit. The wavelength of emitted photon is given by relation,

$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right) \quad n = 5, 6, 7, \dots$$

This series lies in middle Infrared region.

5. Pfund series: Spectral lines of Pfund series are obtained when the electrons make transitions from higher orbits to the 5^{th} orbit. The wavelength of emitted photon is given by relation,

$$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right) \quad n = 6, 7, 8, \dots$$

This series lies in far Infrared region.

27. **State radioactive decay law and hence deduce the expression, $N = N_0 e^{-ht}$.**

Ans: "The rate of radioactive disintegration is directly proportional to the number of radioactive nuclei present in the sample at that instant of time".

Let N be the number of atoms present in a radioactive sample at any instant of time t . If dN is the number of atom disintegrating in a short interval of time dt , then according to decay law

Rate of disintegration \propto number of atoms present

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt \quad \dots \dots \dots (1)$$

Where, λ – decay constant and –ve sign shows that number of radioactive nuclei in the radioactive sample decrease with time.

On integrating the equation (1)

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\ln N = -\lambda t + c \rightarrow (3) \text{ Where 'c' is constant of integration}$$

Let initially at time $t = 0$, the nuclei in the sample $N = N_0$

$$\text{Then } \ln N_0 = -\lambda(0) + c \Rightarrow c = \ln N_0$$

\therefore eqn. (3) changes as

$$\ln N = -\lambda t + \ln N_0 \quad \text{or} \quad \ln N - \ln N_0 = -\lambda t$$

$$\ln \left[\frac{N}{N_0} \right] = -\lambda t$$

taking exponential on both sides

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-ht}$$

28. **Define half life of radioactive sample and obtain the expression for half life. (3/5 M)**

Ans: Half-life of a radioactive sample is defined as 'the time during which number of nuclei in a radioactive sample reduce to half the original value'.

From law of radioactive decay we have, $N = N_0 e^{-\lambda t} \dots \dots \dots (1)$

Where, N – number of radioactive nuclei in the sample at time t

N_0 – number of nuclei at time $t = 0$

λ – decay constant

When, during time $t = T_1$ (half life), number of radioactive nuclei in the sample, $N = \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T_1/2}$$

$$\Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2,$$

taking \log_e on both sides

$$\lambda T_{1/2} = \log_e 2$$

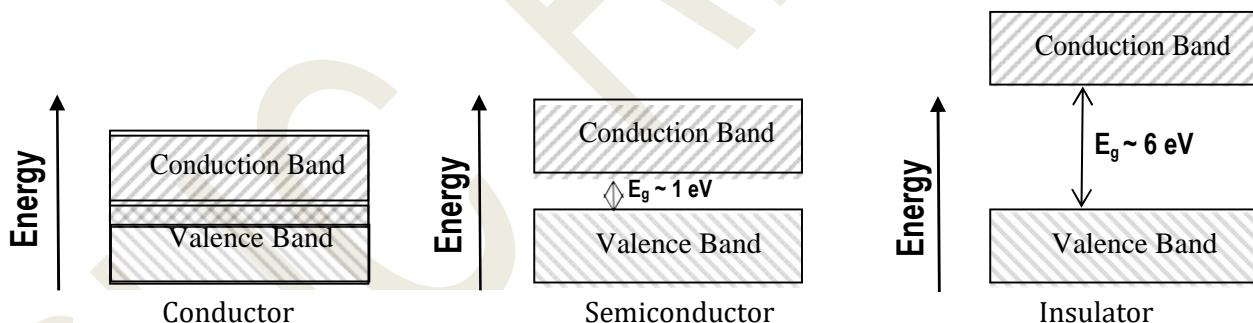
$$T_{1/2} = \frac{0.693}{h}$$

29. Explain the formation of energy bands in solids. Using band theory differentiate between conductors, semiconductors and insulators.

Ans: In an *isolated atom* the electron exist in discrete energy levels. But when the atoms come together to form a solid, the outer orbits of electrons from neighbouring atoms would come very close or could even overlap. Because of this, energy levels of each electron will be very close to each other. The group of such energy levels forming continuous energy variation are called energy bands.

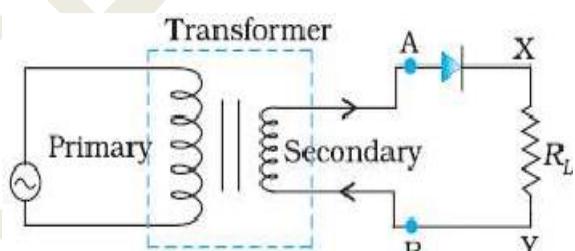
- The energy band which includes the energy levels of the valence electrons is called the **valence band**.
- The energy band above the valence band is called the **conduction band**. The conduction band will be filled by conduction electrons.
- The gap between the top of the valence band and bottom of the conduction band is called the **energy band gap** (Energy gap E_g). It may be large, small, or zero, depending upon the material.

Conductors	Semiconductor	Insulator
Conduction and Valance band are overlapped	Conduction and Valance band are separated by small energy gap ($E_g < 3$ eV)	Conduction and Valance band are separated by large energy gap ($E_g > 3$ eV)
Conduction band is largely filled by conduction electrons	Conduction band is partially filled by conduction electrons	Conduction band is completely empty
Their electrical conductivity is very high	Their electrical conductivity lies between conductors and insulators	The electrical conductivity is not possible.
Their Conductivity <i>decreases</i> with increase in temperature	Their Conductivity <i>increases</i> with increase in temperature	Their Conductivity is <i>independent</i> of temperature
Ex: Metals and their alloys	Ex: Si, Ge	Ex: Plastic, rubber, glass



30. What is half wave rectifier? Describe with a circuit diagram, the working of a diode as half wave rectifier.

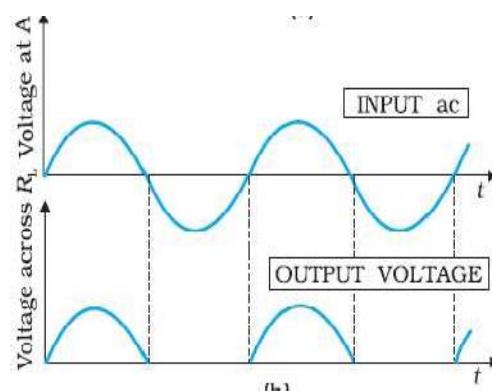
Ans: The device that converts **half cycle of ac** input into **dc** is called half wave rectifier.



- During positive half cycle of the induced ac, the end A of secondary is positive thus the diode D is forward biased. Hence the diode conducts and the output appears across R_L .
- During negative half cycle of the induced ac, the end A of secondary is negative, thus the diode D is reverse biased. Hence the diode do not conducts and no output appears across R_L .
- The cycle of rectifications repeats and the graphical representation of input and out wave forms as in fig.

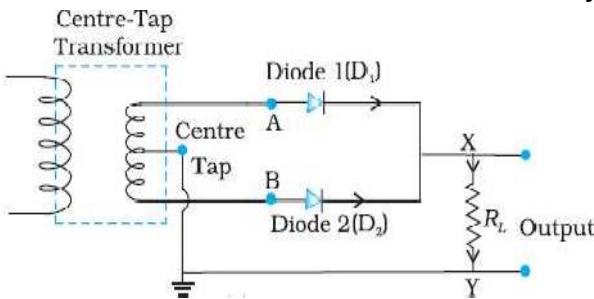
Thus the diode conducts only positive half cycles of input ac cycle and hence it acts as half wave rectifier.

Half wave rectifier circuit is as shown in figure. The ac to be rectified is applied to the primary (P) of the transformer and **induced ac** appears across the secondary. A diode D and the load resistor R_L are connected in series to the free ends of the secondary of the transformer. The output is taken across R_L .



31. What is full wave rectifier? Explain how diodes can be used as a full-wave rectifier.

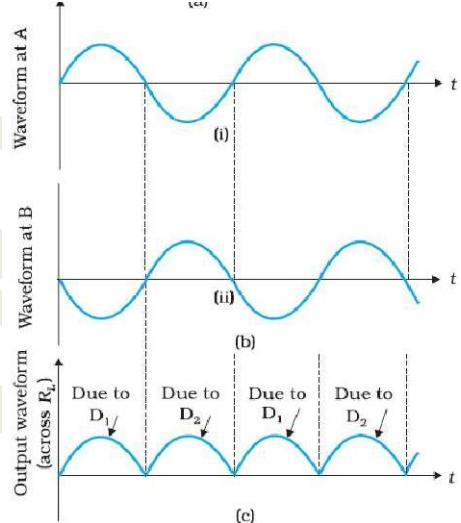
Ans: A device that converts **both the half cycles of ac into dc** is called full wave rectifier.



Full wave rectifier circuit is as shown in figure. The ac to be rectified is applied to the primary (P) of the transformer and the **induced ac** appears across the secondary. The diodes D_1 and D_2 are connected to the free ends of the secondary of the transformer. The negative regions of the diodes are connected to the center tap of the transformer through a load resistance R_L .

- During positive half cycle of the ac input, the end A of secondary is positive relative to center tap and the end B is negative. Thus the diode D_1 is forward biased and D_2 is reverse biased. Hence the diode D_1 conducts and the output appears across R_L .
- During negative half cycle of the ac input, the end A of secondary is negative relative to center tap and the end B is positive. Thus the diode D_1 is reverse biased and D_2 is forward biased. Hence the diode D_2 conducts and the output appears across R_L .
- The cycle of rectifications repeats and the graphical representation of input and out wave forms as in fig.

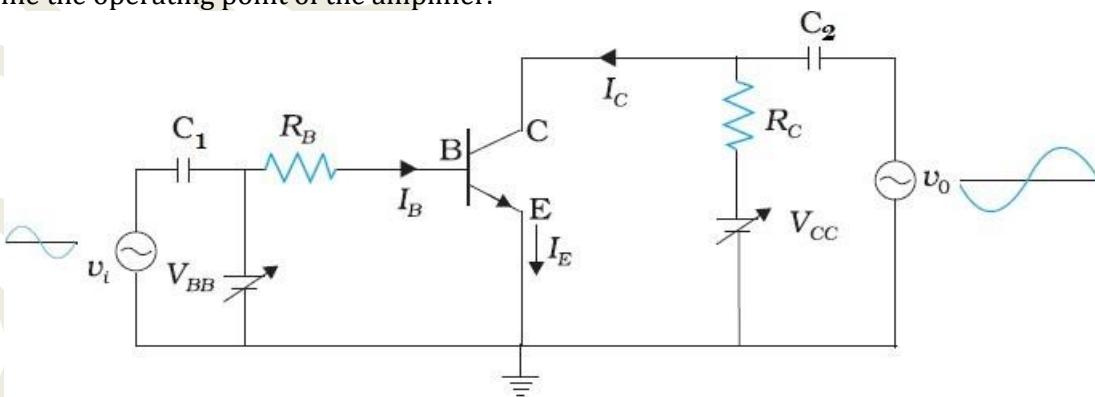
Thus both the halves of input AC is converted into DC and hence the device works as full wave rectifier.



32. Describe with a circuit diagram the working of an amplifier using an npn transistor in CE configuration.

Ans: Amplifier is a device that **increases the amplitude of given signal** and hence increases the strength of the signal.

Transistor as an amplifier is **operated only in active region** of its transfer characteristics. Hence the '**operating voltage**' is fixed in the **middle of the active region**. The operating values of V_{CE} and I_B determine the operating point of the amplifier.



The circuit of CE amplifier employing npn transistor is as shown in figure. Here C_1 and C_2 are coupling capacitors which block DC and allow only AC.

Applying Kirchhoff's laws for input and output circuits, we have

$$V_{BB} = I_B R_B + V_{BE} \quad \dots \dots (1)$$

$$V_{CE} = V_{CC} - I_C R_C \quad \dots \dots (2)$$

A small sinusoidal voltage of amplitude v_i is superposed on the DC base bias V_{BB} (so input voltage = $v_i + V_{BB}$) and output is taken across collector and emitter (output voltage $V_0 = V_{CE}$).

- During the positive half cycle of the input AC signal the emitter-base voltage increases. As a result the input current I_B increases and hence the output current I_C also increases. Consequently the voltage drop across R_C increases. The output voltage V_0 becomes less positive i.e., the amplified output signal goes through a negative half cycle.

- Similarly during negative half cycle of the input AC, input voltage decreases, I_B decreases and I_C also decreases. As a result voltage across R_C also decreases. But the output voltage V_o goes through a positive half cycle. Thus the output voltage $V_o = V_{ce}$ is out of phase by 180° with the input voltage v_i .

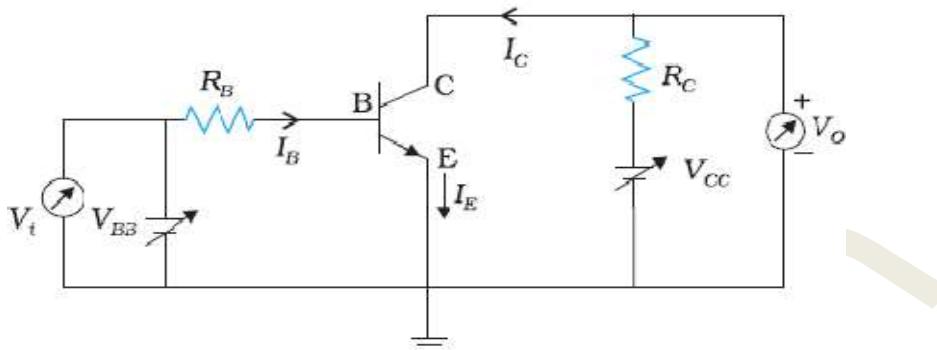
The voltage gain A_v of the amplifier is given by

$$A_V = \frac{-b_{ac}R_C}{r}$$

where, $\beta_{ac} = \frac{I_c}{I_B}$ current amplification factor.

33. Explain the working of transistor as a switch. (3/5 M)

Ans: The npn transistor in CE mode is as shown in figure.



Transistor as a switch **operates only in cut-off state and saturation state without remaining in active state**. The switching circuits are designed in such a way that the transistor does not remain in active state.

Applying Kirchhoff's laws for input and output circuits, we have

$$V_i = V_{BB} = I_B R_B + V_{BE} \dots \dots \dots (1)$$

- When V_i = low i.e less than cut-in voltage (for Si transistor $V_i < 0.6$ V), the input current $I_B = 0$ and consequently output current $I_C = 0$. From equation (2) we have $V_0 = V_{CC}$ and the transistor is **switched off**. The transistor is said to be cut-off state.
- When V_i = high (for Si transistor $V_i > 1$ V), the transistor is in conducting. The output current I_C = maximum and output voltage V_0 = minimum. The transistor is **switched on** and the transistor is said to be in saturation region.

When the transistor is not conducting it is said to be *switched off* and when it is driven into saturation it is said to be *switched on*. This shows that if we define low and high states as below and above certain voltage levels corresponding to cut off and saturation of the transistor, then we can say that a *low* input switches the transistor OFF and a *high* input switches it ON.

IMPT. ADDITIONAL 3/5 QUESTIONS AND ANSWERS:**Sub : Physics****Class : II PUC**

1. Deduce the expressions for radius, angular frequency and time period of circular motion of a charged particle in uniform magnetic field. (3/5 M)

Ans: Consider a particle of mass m charge q entering uniform magnetic field B in a direction perpendicular to the field. The magnetic force makes the path of charged particle circular of radius r in a plane perpendicular to the field. Hence the necessary centripetal force F_C is provided by the magnetic force F_B .

$$F_C = F_B$$

$$\frac{Nv^2}{r} = qvB$$

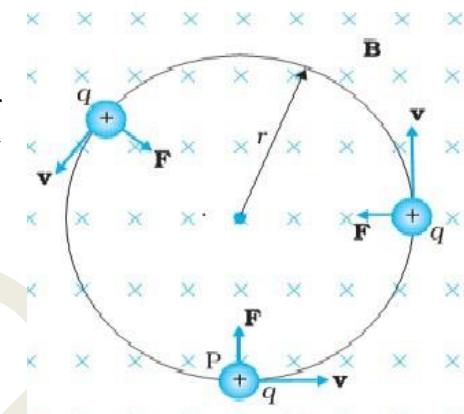
$$\therefore \text{Radius, } r = \frac{mv}{qB}$$

If ω is angular frequency of revolution, then $m = \frac{\omega}{r}$ ($\because v = rm$)

$$\therefore m = \frac{v}{Nv/qB} \Rightarrow m = \frac{qB}{N}$$

Also $m = \frac{2n}{T} = 2nP$, where P is frequency of revolution, $\therefore P = \frac{qB}{2nN}$

$$\text{The time period of charge moving in circular path in magnetic field, } T = \frac{1}{P} \Rightarrow T = \frac{2nN}{qB}$$

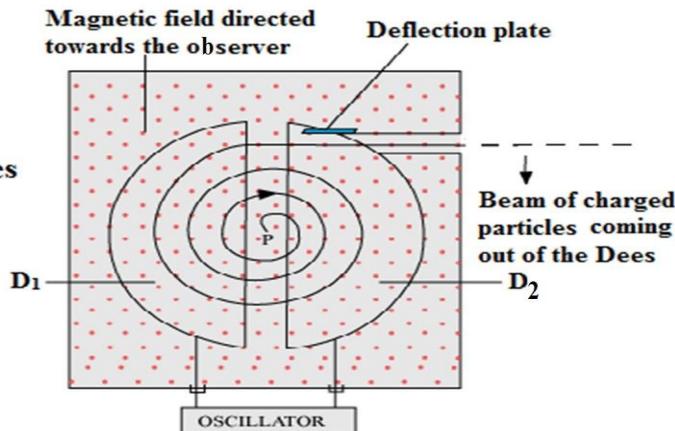


2. With labeled diagram explain the construction and working of a cyclotron.

Ans:

D₁ and D₂-Dees

P- Source of charged particles



Ans: Construction: A cyclotron consists of two hollow semicircular metal discs D_1 and D_2 called Dees. A source of charged particle is located at the midpoint of the gap between the Dees. The Dees are connected to a high frequency oscillator and enclosed inside another vacuum chamber. The whole apparatus is placed between the pole pieces of strong electromagnet.

Working: In cyclotron the charged particle move in presence of both electric and magnetic fields that are perpendicular to each other. The electric field accelerates the charged particle and magnetic field makes its path circular.

The charged particle or ion released from source P accelerates towards the Dee which is at lower potential at that time and describe circular path inside the Dee due to the magnetic field. The frequency of revolution of charged particle inside the cyclotron is called cyclotron frequency, given by

$$P_c = \frac{qB}{2nN}$$

Where, q -charge of charged particle or ion, m -mass of charged particle & B -strength of magnetic field. In cyclotron the frequency of applied ac voltage is made equal to the cyclotron frequency. Hence the charged particle accelerates every time while moving from one Dee to another. As the velocity of charged particle increases, the radius of its circular path also increases. When the radius of circular

orbit of charged particle is nearly equal to the radius of Dee, it is deflected from its path using deflection plate and taken out of the Dee.

The final kinetic energy with which charged particle (ion) comes out from cyclotron is,

$$K = \frac{q^2 B^2 R^2}{2N}$$

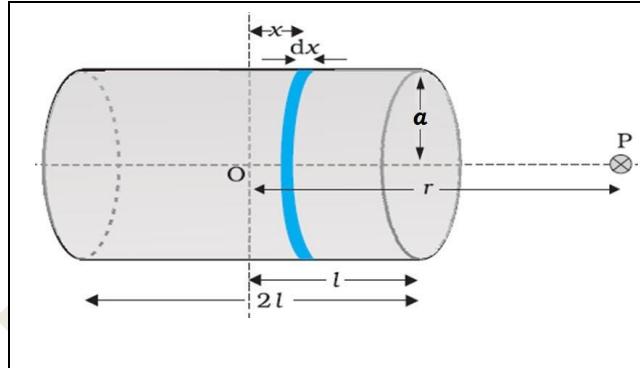
3. Show that the magnetic field at an outside axial point of the current carrying solenoid is equivalent to a bar magnet.

Ans: Consider a solenoid consisting of n turns per unit length carrying a current I . Let its length be $2l$ and radius a .

Consider a circular element of thickness dx of the solenoid at a distance x from the center of solenoid. It consists of ndx turns.

We know that, the magnitude of the magnetic field on the axis of a circular coil is given by,

$$dB = \frac{\mu_0 N I R^2}{2(x^2 + R^2)^{3/2}}$$



Where N is the number of turns in the coil, R is the radius of the coil and x is the distance of the point on the axis of the coil. Here, $N \rightarrow n dx$, $R \rightarrow a$ and $x \rightarrow (r-x)$.

The magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0 n dx I a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements — in other words by integrating from $x = -l$ to $x = +l$. Thus,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{+l} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

Let us consider the point P very far from the solenoid, i.e., $r \gg a$ and $r \gg l$. Then the denominator is approximated by, $[(r-x)^2 + a^2]^{3/2} \approx r^3$ and

$$B = \frac{\mu_0 n I a^2}{2r^3} \int_{-l}^{+l} dx = \frac{\mu_0 n I a^2}{2r^3} [x]_{-l}^{+l} = \frac{\mu_0 n I a^2}{2r^3} [l - (-l)] = \frac{\mu_0 n 2l I a^2}{2r^3}$$

$$B = \frac{\mu_0 2 (n 2l I a^2)}{4n r^3}$$

The magnetic moment of the solenoid is,

$$N = \text{total no. of turns} \times \text{current} \times \text{area of cross section} = (n 2l) I (na^2)$$

$$\therefore B = \frac{\mu_0 2m}{4\pi r^3}$$

This is same as the far axial magnetic field of a bar magnet. Thus, a bar magnet and a solenoid produce similar magnetic fields. Therefore a bar magnet is equivalent to current carrying solenoid.

4. Differentiate between the properties of diamagnetic, paramagnetic and ferromagnetic materials.

Diamagnetic Materials	Paramagnetic Materials	Ferromagnetic Materials
They are weakly repelled by magnetic field.	They are weakly attracted by magnetic field.	They are strongly attracted by magnetic field.
In the external magnetic field, the magnetic field lines are expelled out of these materials. Hence resultant magnetic field inside diamagnetic reduces.	In the external magnetic field, the magnetic field lines are concentrated in these materials. Hence resultant magnetic field inside paramagnetic increases.	In the external magnetic field, the magnetic field lines are highly concentrated in these materials. Hence resultant magnetic field inside ferromagnetic increases.

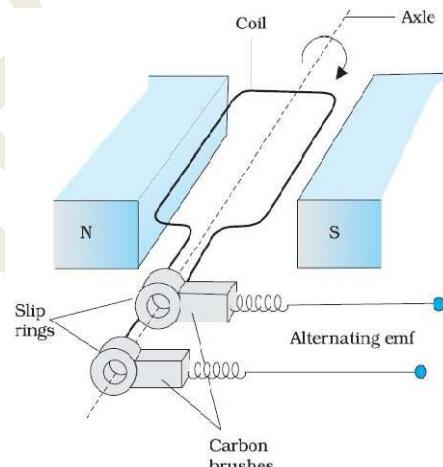
Their magnetic susceptibility is low and negative ($\chi < 0$).	Their magnetic susceptibility is low and positive ($\chi > 0$).	Their magnetic susceptibility is high and positive ($\chi \gg 0$).
Their relative permeability is less than one ($\mu_r < 1$)	The relative permeability is greater than one ($\mu_r > 1$)	The relative permeability is much greater than one ($\mu_r \gg 1$)
Their magnetic susceptibility (3) is independent of temperature.	Their magnetic susceptibility (3) decreases with increase in temperature.	Their magnetic susceptibility (3) decreases with increase in temperature and at certain high temperature they become paramagnetic.

5. **What is an ac generator? Give the principle of generator. Derive the expression for generation of sinusoidal emf by an ac generator. Draw the graph showing the variation of the induced emf with time.**

Ans: An ac generator is a device which converts mechanical energy into electrical energy.

Principle: One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. As the coil rotates in a magnetic field \vec{B} , the effective area of the loop (the face perpendicular to the field) is $A \cos \theta$, where θ is the angle between \vec{A} and \vec{B} . This method of producing a flux change is the principle of operation of a simple ac generator.

Construction: It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.



Working/Theory: When the coil is rotated with a constant angular speed ω , the angle θ between the magnetic field vector \vec{B} and the area vector \vec{A} of the coil at any instant t is $\theta = \omega t$ (assuming $\theta = 0^\circ$ at $t = 0$). As a result, the effective area of the coil exposed to the magnetic field lines changes with time, and the flux at any time t is

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \cos \omega t$$

From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$s = -N \frac{d\Phi_B}{dt} = NBA \frac{d}{dt} (\cos \omega t)$$

Thus, the instantaneous value of the emf is

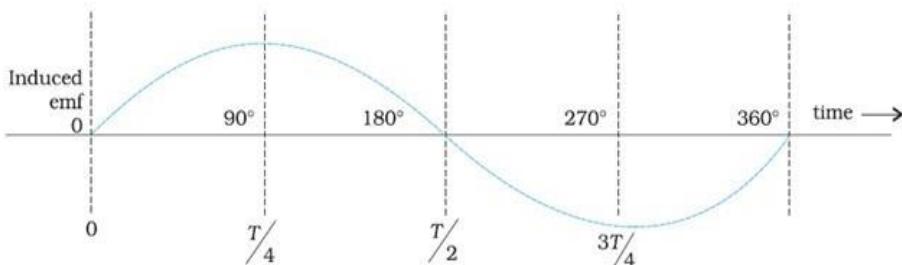
$$s = NBA \omega \sin \omega t$$

Where, $NBA\omega$ is the maximum value of the emf, which occurs when $\sin \omega t = \pm 1$ and is denoted as s_0 , then $s = s_0 \sin \omega t$

The direction of the emf and hence current changes periodically and therefore the current is called *alternating current (ac)*.

Since $\omega = 2\pi v$, then we can write, $s = s_0 \sin 2\pi v t$

where v is the frequency of revolution of the generator's coil.



An alternating emf is generated by a loop of wire rotating in a magnetic field.

6. Explain the construction, working and theory of a transformer.

Ans: A transformer is a device used to vary (step up or step down) AC voltages. It works on the principle of mutual induction.

Construction

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, either one on top of the other as in Fig. (a) or on separate limbs of the core as in Fig. (b). One of the coils called the *primary coil* has N_p turns. The other coil is called the *secondary coil*; it has N_s turns. The primary coil is the input coil and the secondary coil is the output coil of the transformer.

Working

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it. The value of this emf depends on the number of turns in the secondary.

Theory of transformer/Expression for turns ratio

Consider an ideal transformer in which the primary has negligible resistance and all the flux in the core links both primary and secondary windings.

Let ϕ be the flux in each turn in the core at time t due to current in the primary when a voltage v_p is applied to it. Then the induced emf or voltage ε_s in the secondary with N_s turns is

$$s_c = -N_s \frac{d\phi}{dt} \quad \dots \quad (1)$$

The alternating flux ϕ also induces an emf, called *back emf* in the primary. This is

$$s_p = -N_p \frac{d\phi}{dt} \quad \dots \quad (2)$$

But $s_p = v_p$. If this were not so, the primary current would be infinite since the primary has zero resistance (as assumed). If the secondary is an open circuit or the current taken from it is small, then to a good approximation $s_c = v_s$

where v_s is the voltage across the secondary. Therefore, Eqs. (1) and (2) can be written as

$$v_s = -N_s \frac{d\phi}{dt} \quad \dots \quad (3)$$

$$v_p = -N_p \frac{d\phi}{dt} \quad \dots \quad (4)$$

Dividing eq. (3) by (4), we have

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} = T$$

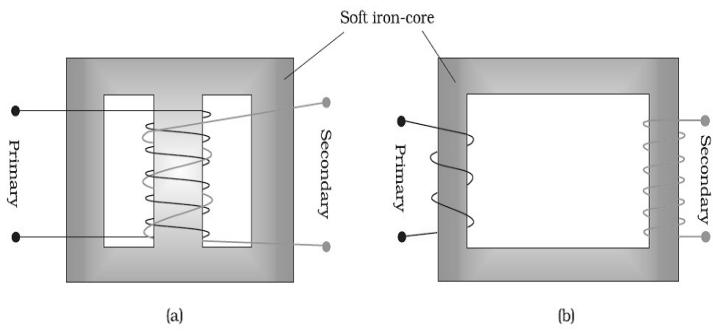
and is called turns ratio.

- If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output, and since $p = iv$, $\therefore i_p v_p = i_s v_s \Rightarrow \frac{i_p}{i_s} = \frac{v_s}{v_p}$

Step-up transformer: For step up transformer, turns ratio $T > 1$, that is, the secondary coil has a greater number of turns than the primary ($N_s > N_p$), the voltage is stepped up (as $V_s = (\frac{N_s}{N_p}) V_p$ and hence $V_s > V_p$). This type of arrangement is called a *step-up transformer*. However, in this arrangement, there is less current in the secondary than in the primary ($N_p/N_s < 1$ and $I_s < I_p$).

Step-down transformer

For step down transformer, turns ratio $T < 1$, that is the secondary coil has less turns than the primary ($N_s < N_p$), we have a *step-down transformer*. In this case, $V_s < V_p$ and $I_s > I_p$. That is, the voltage is stepped down, or reduced, and the current is increased.



7. Using Huygens principle show that angle of incidence is equal to the angle of reflection for a reflection of plane wave front at a plane surface.

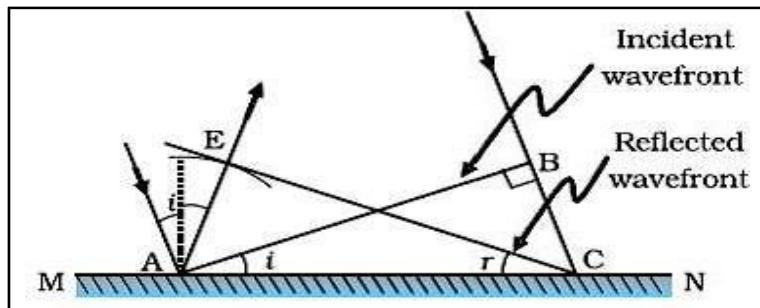
Ans:

MN – reflecting surface

AB – Incident plane wave front

i – angle of incidence

v – speed of light in the medium



If v represents the speed of the wave in the medium and if τ represents the time taken by the wave front to advance from the point B to C then the distance BC is, $BC = v\tau$ ----- (1)

In order to construct the reflected wave front, a sphere of radius $= v\tau$, is drawn from the point A as shown in the adjacent figure. The tangent plane CE drawn from the point C to this sphere represents reflected wave front.

$\therefore AE = BC = v\tau, \angle ABC = \angle CEA = 90^\circ, AC$ is common.

Triangles EAC and BAC are congruent.

$\therefore i = r$ ----- (2)

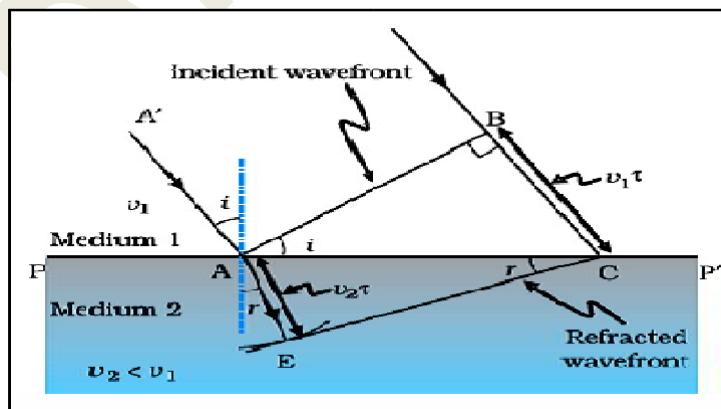
This is the *law of reflection*.

8. Explain the refraction of plane waves using Huygens principle and hence arrive at Snell's law of refraction. (5 M)

Ans:

PP' - surface separating medium-1 and medium-2

v_1 and v_2 - speed of light in medium-1 and medium-2 respectively.



Consider a plane wave front AB incident in medium-1 at angle 'i' on the surface PP' . According to Huygens principle, every point on the wave front AB is a source of secondary wavelets.

Let the secondary wavelet from B strike the surface PP' at C in time τ .

Then $BC = v_1\tau$.

The secondary wavelet from A will travel a distance $v_2\tau$ as radius; draw an arc in medium 2. The tangent from C touches the arc at E. Then $AE = v_2\tau$ and CE is the refracted wave front. Let r be the angle of refraction.

In the figure, $\angle BAC = i =$ angle of incidence and $\angle ECA = r =$ angle of refraction

$$BC = v_1\tau \text{ and } AE = v_2\tau$$

$$\text{From triangle BAC, } \sin i = \frac{BC}{AC} \text{ and from triangle ECA, } \sin r = \frac{AE}{AC}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{BC/AC}{AE/AC} = \frac{BC}{AC} = \frac{v_1 v}{v_2 v} = \frac{v_1}{v_2} \quad \text{--- (1)}$$

Now, refractive index (n) of a medium: $n = \frac{c}{v}$ where c - speed of light in vacuum.

$$\text{For the first medium: } n_1 = \frac{c}{v_1} \quad \text{and for the second medium: } n_2 = \frac{c}{v_2} \Rightarrow \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

$$\text{Equation (1) becomes, } \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad \text{or} \quad n_1 \sin i = n_2 \sin r$$

This is the **Snell's law of refraction**.

9. Give the theory of interference and hence arrive at the conditions for constructive and destructive interference.

Ans: Consider two coherent sources of light S_1 and S_2 .

Let the displacement produced by source S_1 is, $y_1 = a \cos(\omega t)$
and the displacement produced by source S_2 is, $y_2 = a \cos(\omega t + \phi)$
where, a - amplitude of waves

ϕ - phase difference between the waves.

The resultant displacement is, $y = y_1 + y_2$

$$y = [a \cos(\omega t) + a \cos(\omega t + \phi)]$$

$$y = a [\cos(\omega t) + \cos(\omega t + \phi) + \cos(\omega t) + \cos(\omega t + \phi)]$$

$$y = 2a \cos\left(\frac{\phi}{2}\right) \cos\left(\frac{\omega t + \phi}{2}\right)$$

$$y = R \cos\left(\frac{\omega t + \phi}{2}\right) \quad (1)$$

$$\text{Using } \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\text{The amplitude of the resultant displacement is } R = 2a \cos\left(\frac{\phi}{2}\right) \quad (2)$$

The intensity at a point is, Intensity \propto (amplitude)²

$$\therefore \text{The intensity at that point will be, } I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \quad (3)$$

Where, I_0 - intensity of interfering individual wave

Conditions for Constructive Interference:

For the constructive interference at an arbitrary point P, the intensity of light at that point is to be

maximum, i.e. $I_{\max} = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = +1 \Rightarrow \cos^2\left(\frac{\phi}{2}\right) = 1 \Rightarrow \cos\left(\frac{\phi}{2}\right) = \pm 1$

OR phase difference, $\phi = 0, \pm 2\pi, \pm 4\pi, \dots$ Or $\phi = \pm n\pi$

OR path difference: $[\delta = n\lambda]$, (Where $n = 0, 1, 2, 3, \dots$)

Conditions for Destructive Interference:

For the destructive interference at an arbitrary point P, the intensity of light at that point is to be

minimum, i.e. $I_{\min} = 0 \Rightarrow \cos^2\left(\frac{\phi}{2}\right) = 0 \Rightarrow \cos\left(\frac{\phi}{2}\right) = 0$

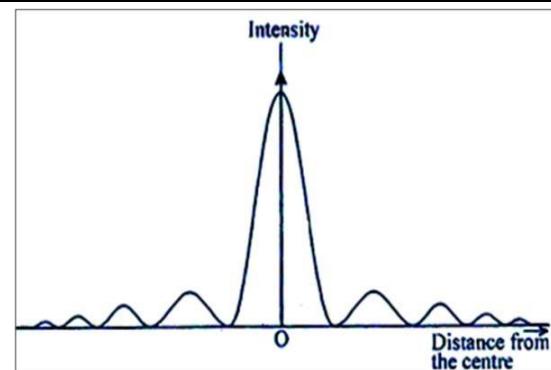
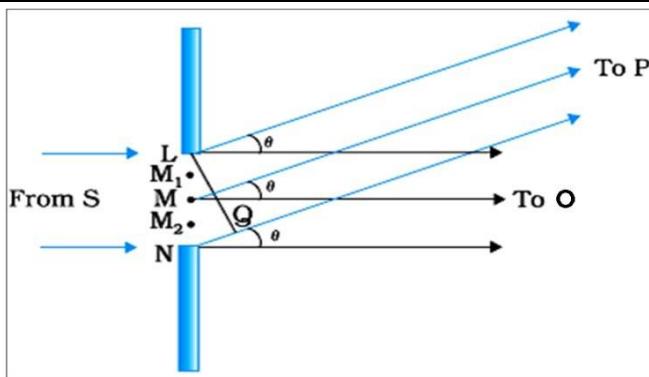
OR phase difference, $\phi = \pm \pi, \pm 3\pi, \pm 5\pi, \dots$ Or $\phi = \pm (2n + 1)\pi$

OR path difference: $[\delta = (n + \frac{1}{2})\lambda]$, (Where $n = 0, 1, 2, 3, \dots$)

10. What is diffraction of light? Explain diffraction of light at single slit. (3/5 M)

Ans: The phenomenon of bending of light waves around the edges (or corners) of the obstacles and entering into the expected geometrical shadow of the obstacle is called **diffraction of light**.

Consider a parallel beam of light falling normally on a single narrow slit LN of width a . The different parts of the wave front at the slit act as secondary sources of disturbance. The secondary waves (diffracted light) from the slit, interfere producing diffraction pattern on the screen.



When single narrow slit illuminated by a monochromatic source, a broad pattern with a **central bright region** is seen. On both sides of central fringe, there **are alternate dark and bright regions**; the **intensity becomes weaker** away from the centre, as shown in intensity distribution curve.

The path difference between the two edges of the slit L and N at a point P on screen is,

$$NP - LP = NQ = a \sin \theta \approx a\theta$$

Formation of central fringe:

At the central point O on the screen, secondary waves from corresponding parts of the slit arrive in phase (the all path differences of secondary waves are zero) and this gives maximum intensity at O. Hence the intensity has a central maximum at $\theta = 0$.

Conditions for Secondary maxima:

The Secondary maxima are produced at $\theta \approx \left(n + \frac{1}{2} \right) \frac{\lambda}{a}, n = \pm 1, \pm 2, \pm 3, \dots$

For first secondary maximum: $n=1$, this implies, $\theta \approx \frac{3}{2} \left(\frac{\lambda}{a} \right)$

Condition for minima: The Minima (zero intensity) is produced at $\theta \approx \frac{n\lambda}{a}, n = \pm 1, \pm 2, \pm 3, \dots$

11.

Define Brewster's angle and hence arrive at the Brewster's law of polarisation.

Ans: The tangent of the Brewster's angle (i_B) is equal to refractive index (n) of the material of the reflector, i.e., $n = \tan i_B$

When the angle of incidence on a surface is equal to the Brewster's angle, **the reflected and refracted rays are perpendicular** to each other.

Proof: Consider a beam of an unpolarised light incident at an angle equal to the Brewster's angle i_B on the surface of a transparent material having refractive index n.

PO is the ray of light incident on the surface of glass slab of refractive index n at an angle i_B , the Brewster's angle. OQ is the reflected ray, which is completely polarized. OR is the refracted ray and is partially polarised. Let r be the angle of refraction.

It is observed that the **reflected and the refracted rays are perpendicular** to each other.

$$\text{i.e. } \angle QOR = 90^\circ \quad (1)$$

From laws of reflection, $\angle POM = \angle MOQ = i_B$

From the figure,

$$\angle MOQ + \angle QOR + \angle RON = 180^\circ$$

$$i_B + 90^\circ + r = 180^\circ$$

$$\Rightarrow r + i_B = 90^\circ$$

$$\Rightarrow r = 90^\circ - i_B \quad (2)$$

$$\text{From Snell's law, } n = \frac{\sin i}{\sin r} = \frac{\sin i_B}{\sin r}$$

$$\Rightarrow n = \frac{\sin i_B}{\sin (90^\circ - i_B)} = \frac{\sin i_B}{\cos i_B}$$

$$\Rightarrow n = \tan i_B$$

This is **Brewster's law**.

